
The Works of Archimedes: Volume I. The Two Books On the Sphere and the Cylinder by Reviel Netz

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This is the initial volume in a proposed project to supply the first English translation of the complete works of Archimedes that survive in Greek. This volume is based on the text of Heiberg's edition as revised by Stamatis [Heiberg and Stamatis 1972]. The other volumes await the new edition of the Archimedes Palimpsest now in progress [2].¹ This project, and the careful scholarship Netz brings to it, will be a most welcome addition to our understanding of the mathematics and exact sciences of the Hellenistic period.

In this volume, Netz provides a translation of, and commentary on, Archimedes' *On the Sphere and the Cylinder* (*SC*) as well as a translation of the *Commentary* to it made by the Byzantine scholar, Eutocius of Ascalon. The two books of *SC* were originally published in the form of open letters sent separately to a certain Dositheus.² The first book develops a general theory of the metrical properties of

¹ See <http://www.thewalters.org/archimedes/frame.html> for a brief overview of the story of this palimpsest.

² Netz believes that these two books were originally separate treatises, which were then put together by some later editor [19]. His claim that they are each a self-contained essay is difficult to understand with respect to *SC* 2. It makes repeated use of high-level theorems from *SC* 1 of the sort that Greek geometers almost never use without first proving. Although *SC* 1 is more self-contained in the sense that it comes first, it also bears a clear mathematical relation to *SC* 2. Despite the fact that many theorems in *SC* 1 are inherently interesting, the book as a whole is motivated by the use to which it will be put in *SC* 2. Moreover, Netz' position compels him to argue for the systematic excision of references to the first book which are found in *SC* 2. This is the only case where Netz wants to apply a general principle of removing text that Heiberg found satisfactory.

geometrically related spheres, cylinders, and cones. The second then uses this theory to solve a number of problems and to demonstrate a few theorems involving these same objects.

The 44 propositions of *SC 1* take up over twice as many pages as the nine propositions of *SC 2*; but quantity is no substitute for quality. Whereas *SC 2* contains some of the most impressive Greek mathematics we possess, much of *SC 1* is mathematically simplistic. There is considerable repetition and minor variation; and we sense Archimedes' boredom as he rushes along, too annoyed with such trivialities to waste time with undue rigor. There are brilliant results in this book, but even these seem almost to be afterthoughts in Archimedes' presentation. In some ways, *SC 2* presents us with the opposite situation. This is advanced mathematical research, and we feel as though we are watching Archimedes venturing out alone into uncharted lands and seeing for the first time a strange new world. By the end of the book, even his means of expression have become innovative.

Eutocius' *Commentary* reflects this basic division in the text. The *Commentary* to *SC 1* is short and largely trivial. After an interesting discussion of the definitions, the book is just a series of elementary proofs providing justifications for steps in *SC 1* that Archimedes considered too elementary to warrant full justification. The much longer *Commentary* to *SC 2*, however, contains a considerable amount of exciting mathematics. This extra length is primarily due to two long digressions that give us important insight into some of the more advanced mathematical methods of the Classical and Hellenistic periods. There is also an interesting section in which Eutocius advances his own contributions to the theory of compound ratio. Netz suggests that the difference between the two commentaries is due to the fact that Eutocius had matured between their compositions [312n299], but I suspect it has more to do with the latter's interest in the mathematics involved. The preponderance of problems in *SC 2* gave Eutocius occasion to situate Archimedes' work in the rich tradition of geometric problem-solving, a tradition in which we find many of the great names in Greek mathematics. Moreover, the level of mathematics in this book is generally higher; and Eutocius no doubt felt that it gave him more opportunity to show his caliber, both as scholar and as a mathematician.

After a short introduction, Netz' study of Archimedes proceeds by way of: (1) the translation itself, (2) critical diagrams, (3) textual commentaries, and (4) general commentaries. There is no mathematical commentary and, given the nature of the text, there are places where this absence is conspicuous. For Eutocius' commentary, Netz does not provide textual and general commentaries on each theorem, although the footnotes are generally fuller.

The Translation

The translation itself often makes for difficult reading because it tries to reproduce in English something linguistically similar to what we find in the Greek. English and Greek, however, are very different languages. Indeed, reading Netz' translation did simulate, to some extent, that uncanny feeling that I had the first time I turned my attention to Archimedes' prose and before I had read Heath's very useful chapters on the linguistic practices of Greek mathematicians [Heath 1896, cclvii-clxx; 1912, clvi-clxxxvi]. As Netz' first book [1999] so aptly demonstrates, however, Greek mathematicians use specific features of the Greek language to streamline their texts and to keep the reader's mind focused on the mathematical objects at issue. Many of these features cannot be reproduced with the same effect in English, and the resulting translation is often strange. Netz acknowledges this problem in his introduction and admits that in some places 'the English had to give way to the Greek' [3].

There are many places where I felt the translation was unduly literal. A few examples will make the point. Netz translates every definite article in Greek with a definite article in English, despite the fact that the expression 'line AB ' or simply ' AB ', for example, is already suitably definite in English, being a title and a proper name.³ In one case, Netz tries to reproduce a Greek idiom meaning 'one and the other' by a repetition of the same word. This yields the translation 'the perpendicular drawn from the vertex of the other

³ Overliteral translation of the definite article can sometimes yield a misleading sense. For example, Netz translates $\kappa\alpha\iota\ \beta\acute{\alpha}\sigma\iota\nu\ \mu\acute{\epsilon}\nu\ \xi\chi\epsilon\iota\ \tau\acute{\alpha}\ \tau\acute{\rho}\iota\gamma\omega\nu\alpha\ \tau\acute{\alpha}\zeta\ AB, B\Gamma, \Gamma A$ by 'And the triangles have <as> base the <lines> $AB, B\Gamma, \Gamma A$ ' [57]. The text, however, simply means, 'The triangles have base $AB + B\Gamma + \Gamma A$.'

cone to the side of the other cone' [105]. Given that there are only two cones involved, this phrase is peculiar and possibly meaningless. Netz tries, as far as possible, to translate individual words consistently. In the case of prepositions, this naturally creates some strained passages. For example, in the enunciations of *SC* 1.37 and 1.38, Netz speaks of lines being drawn from the vertex of one object *on* ($\acute{\epsilon}\pi\acute{\iota}$) another object, whereas Archimedes clearly means from the first object *to* the second [158, 160].⁴

Overall, the translation is technically proficient; however, there are a few slips. In order to keep the reader constantly mindful of the strong tendency of Greek mathematical prose toward ellipsis, Netz supplies the words, missing in Greek, between angle brackets, $\langle \dots \rangle$. Sometimes, however, the wrong word gets into these brackets. For example, ' \langle the lines \rangle ', in the enunciation of *SC* 1.12, should almost certainly refer to the aforementioned *tangents* [77]. In the exposition of *SC* 1.42, the gender of the article and the mathematical conditions both argue that the text means 'line AF '; whereas Netz translates it by 'the \langle diameter \rangle AF ' [174].

There are other minor mistakes that have little effect on the mathematical sense. For example, $\delta\iota\lambda\acute{\alpha}\ \tau\tilde{\eta}\varsigma\ AF\ \acute{\epsilon}\pi\iota\pi\acute{\epsilon}\delta\omega$, which means 'by a plane through line AF ', is translated as 'by the plane AF ' [202]. In some cases, however, the mathematical sense is affected. Thus, $\acute{\epsilon}\pi\iota\pi\acute{\epsilon}\delta\omega\ \acute{\omicron}\rho\theta\tilde{\omega}\ \tau\tilde{\omega}\ \kappa\alpha\tau\acute{\alpha}\ \tau\tilde{\eta}\nu\ A\Delta$, which means 'by a plane orthogonal with respect to line $A\Delta$ ', has been translated as 'by a plane \langle which is \rangle right to the \langle plane \rangle at $A\Delta$ ' [177]. And $\kappa\alpha\iota\ \acute{\epsilon}\kappa\beta\epsilon\beta\lambda\acute{\eta}\sigma\theta\omega\ \pi\rho\acute{\omicron}\varsigma\ \tau\tilde{\eta}\nu\ AB\ \acute{\epsilon}\pi\acute{\iota}\pi\epsilon\delta\omicron\nu\ \acute{\omicron}\rho\theta\acute{\omicron}\nu$ means 'Let a plane orthogonal to line AB be produced'; whereas Netz has 'Let a right plane be produced, \langle in right angles \rangle to AB ' [199].⁵

Netz chooses to translate all of the operations on proportions by means of adverbs. Two of the adverbs he uses for this are, in

⁴ In fact, Netz, translates $\acute{\epsilon}\pi\acute{\iota}$ in *SC* 1.39 more naturally by 'to' [163]. Perhaps the earlier two occurrences of 'on' are typos.

⁵ Here Netz states: 'In itself this does not say much. The idea is for the plane to be right to the great circle that passes through AB ' [199n71]. It is not clear which great circle he means. At any rate, most of the planes perpendicular to most of these great circles are not the ones Archimedes intends. Archimedes simply means a plane which is perpendicular to line AB .

my view, unfortunate.⁶ He uses ‘compoundly’ for the operation that Heath translates by ‘*componendo*’ or ‘composition’, that is

$$A : B = C : D \rightarrow (A + B) : B = (C + D) : D.$$

In almost all English secondary literature on Greek mathematics, however, the ratio $A : B$ is said to be the compounded of the ratios $C : D$ and $E : F$ when

$$A : B = (C : D) \times (E : F).$$

Netz himself generally uses ‘combined’ or a cognate to refer to *compound ratios*. Although the Greek words used for these two operations are cognates, they denote very different operations, and I am not aware of any cases where it is ambiguous which operation the geometer intends. It seems needlessly confusing to start switching the two now that there is an established and useful practice.⁷

Furthermore, Netz uses ‘dividedly’ for the operation that Heath translates by ‘*separando*’ or ‘separation’, that is,

$$A : B = C : D \rightarrow (A - B) : B = (C - D) : D,$$

where $A > B$ and $C > D$. Since this operation has nothing to do with what we mean when we generally speak of division in a mathematical context, Netz’ expression is misleading.⁸ This becomes most pronounced when he translates $\kappa\alpha\iota \gamma\acute{\alpha}\rho \tau\grave{\alpha} \kappa\alpha\tau\grave{\alpha} \delta\iota\alpha\acute{\iota}\rho\epsilon\sigma\iota\nu$ as ‘for the <things shown> according to division, too’ [215]. $\kappa\alpha\tau\grave{\alpha} \delta\iota\alpha\acute{\iota}\rho\epsilon\sigma\iota\nu$ is a technical expression in both logic and mathematics. In later

⁶ For operations on ratios, I follow the terminology standardized by Heath [1956].

⁷ Netz himself acknowledges the confusion in a footnote. In Eutocius’ commentary to *SC* 2.4, we encounter the expression $\delta\iota\grave{\alpha} \tau\omicron\upsilon \sigma\upsilon\nu\theta\acute{\epsilon}\nu\tau\iota$ which probably means something like ‘through the operation of composition’ (literally, ‘through composition’). Netz translates this as ‘through the “compoundly”’; and appends a note which reads, ‘This time “compoundly” refers not to the composition-of-ratios operation, but to the “compoundly” proportion argument, *Elements* V.18’ [316n325]. Here, apparently, Netz is referring to what everyone else calls ‘compound ratio’ as the ‘composition-of-ratios operation’. Moreover, I cannot find any previous passage in his book which uses ‘compoundly’ to refer to the ‘composition-of-ratios operation’.

⁸ This issue has already been raised by Heath [1956, 2.135], in making his case for ‘separation’.

mathematical writers, it has the same meaning as διελόντι, the dative of means which is generally used for the operation of separation. This cryptic reference points to the fact that separation is the opposite operation to composition. Netz' footnote makes this clear, but his translation confuses the issue [215n156].

The overall method of Netz' translation raises a number of interesting questions concerning the goal and methodology of translation in general. Recent books by Jens Høyrup [2002: cf. Steele 2004] and Netz himself [1999] have contributed greatly to our understanding the methods of ancient mathematical traditions by producing translations and commentaries that stay very close to the original languages. These translations help to reveal the conceptual contexts in which ancient mathematics was practiced, but they make for trying English. This linguistic difficulty, however, is mitigated by the fact that the translations are set in an interpretive framework that makes their value clear and immediate.

It seems that Netz has now turned these principles to making a general translation, a reader's text. Netz claims that the purpose of a scholarly translation 'is to remove all barriers having to do with the foreign language itself, leaving all other barriers intact' [3]. Perhaps this is so, but the Greek mathematicians employed the particularities of their language in many ways that cannot be effectively reproduced in English. My concern is that English readers who are unfamiliar with those features of the Greek language that make its mathematical prose so effective, may come away with the impression that Archimedes did not know how to write. For example, the tendency toward ellipsis gives the articles and prepositions an abbreviating function such that the text stays focused on the mathematical objects, not cluttered with unnecessary verbiage. The statements are primarily about lettered objects. Given a passage such as

ὁ δὲ τοῦ ἀπὸ $A\theta$ πρὸς τὸ ἀπὸ θB προσλαβῶν τὸν τῆς $A\theta$
πρὸς θB ὁ τοῦ ἀπὸ $A\theta$ ἐστὶν πρὸς τὸ ὑπὸ τῶν $\Gamma\theta B$,

I wonder whether Heiberg's

$$(A\theta^2:\theta B^2) \times (A\theta:\theta B) = A\theta^2:(\Gamma\theta \times \theta B)$$

is not as close to Archimedes' style as Netz'

but the <ratio> of the <square> on $A\theta$ to the <square> on θB , taking in the <ratio> of $A\theta$ to θB , is the <ratio> of the <square> on $A\theta$ to the <rectangle contained> by $\Gamma\theta B$.

on p. 229. Of course, there are no symbols in the Greek, but neither are there any nouns.

In a number of places, Netz makes interesting comments that are supported by the proximity of his translation to the original Greek. I wonder, however, whether this could not also be done following a more accessible translation, simply by giving a second, more literal translation in the few places where this is really necessary.

The Critical Diagrams

Netz has redrawn all of the diagrams based on a new examination of the principal manuscripts. These diagrams are accompanied by a critical apparatus. This constitutes the first critical edition of the manuscript figures and should be welcomed as an important contribution to scholarship, both in terms of its results and its methodology. Moreover, it means that we now have general access to the figures of the manuscript tradition and quite possibility to figures which approach those drafted by Archimedes to accompany his text.

As Netz [1999] has shown, the medieval diagrams should be studied as an important, and in some sense independent, window on Greek mathematical practice. Although he does not give a full treatment of the figures in this work, Netz makes a number of interesting comments about them. For example, he points out the relationship between objects that are actually constructed in the diagram and objects that are invoked through the operation of imagination. Objects are imagined when they cannot be adequately or suitably represented in the figure. Nevertheless, once these objects are so imagined, they can then be used in the course of the argument in much the same way as objects which have been more straightforwardly constructed. Again, Netz underlines the schematic nature of the diagrams. Greek diagrams are not meant to depict the mathematical objects visually, but to represent certain logical or structural elements, features that we might call topological.

Because of the scattered nature of Netz' remarks on the diagrams, it is difficult to state precisely his account of Greek mathematical thought with regard to diagrams. By my reading, however, there appears to be an inconsistency in two positions that he holds. In the introduction, he asserts that 'Greek mathematical proofs always refer to concrete objects, realized in the diagram.' This seems true, but our interpretation of this statement will depend on exactly how we understand the realization that the diagrams achieve. On the one hand, Netz states that, for Greek geometers, 'the diagram was the actual mathematical object,' and implies that geometric discourse is primarily about this diagrammatic object [81]. In this vein he also speaks of a 'diagrammatic reality' [76]. On the other hand, he believes that the diagrams 'provide a schematic representation of the pattern of configuration holding in the geometrical case studied' [9: cf. 46, 107]. In one case, he refers to a 'geometric reality' which is in fact metrically divergent from the representation in the diagrams [101]. If, in fact, the diagrams are schematic, then they must represent the organizational structure of some more fundamental objects. That is, the diagrams must point toward the objects of discourse in the same way as the text; they cannot themselves constitute this object.

The Commentaries

Netz provides both a textual and a general commentary to each unit of Archimedes' text. The textual commentaries give a very useful discussion of the many issues arising out of the vagaries of manuscript practices. The general commentaries are more conceptual and literary reflections on Archimedes in particular and on Greek mathematics in general.

The text of *SC* appears to have undergone considerable editorial intervention. At the most basic level, the dialect has been modified from Archimedes' native Doric into the common dialect of the Hellenistic and later periods. At a more mathematically significant level, there are many demonstrable insertions, some originating from Eutocius' *Commentary*, but others probably having entered the text before Eutocius' time. This state of affairs prompted Heiberg to form an idealized notion of Archimedes as a prose stylist and to tag as insertions any bit of text that did not meet his, sometimes vaguely defined, criteria. Netz' Textual Comments are helpful in a number

of ways: they discuss the difficulties associated with assuming that all of the text was written by Archimedes, they question a number of Heiberg's presumptions concerning Archimedes' prose style, and they make strong arguments for those passages which almost certainly are interpolations. These comments also give good treatment to a number of localized issues as they arise.

The General Commentary collects any remarks that are not of a strictly textual nature. Here we find remarks on the mathematical methods, logical structures, conceptual contexts, and rhetorical strategies with which Archimedes works. In many ways, these commentaries will constitute Netz' most innovative contribution to Archimedes scholarship. They keep the reader mindful of the rhetorical forms that Archimedes employs and of how these can be meaningfully interpreted in the context of other Greek mathematical writings. They underline the many features of Archimedes' text that are specific to it as an act of communication. Many of Netz' most interesting observations are reiterations, or extensions, of findings in his *Shaping of Deduction*. I imagine that most readers will find these sections both interesting and challenging.

Because of the interpretive nature of these comments, there are quite a few places where I do not agree with Netz' reading. Probably, in many cases, good arguments could be made for either view; nevertheless, because they have implications concerning how we understand Greek mathematics in general, it seems appropriate to present a few examples.

Netz believes that Greek mathematicians tend to conflate equality and identity; whereas it seems to me that, by and large, they differentiate between the two. In fact, the reflective property of equality, $x = x$, is a fairly abstract notion. Greek mathematicians talk about a line being equal to another line, but about a ratio being the same as another ratio. They mean this quite literally. The lines are different lines but equal in length; the ratios, in contrast, are two instantiations of the very same ratio. Generally, metrical properties can be abstracted from the objects themselves, but ratios are not metrical properties that belong to a single object. Netz argues against this position and refers to texts like

τριγώνω βάσιν μὲν ἔχοντι τὴν ἴσην ταῖς $AB, BF, ΓA$ ὕψος δὲ
τὴν εἰρημένην εὐθεΐαν,

which he translates as

to a triangle having a base equal to AB, BF, FA and, <as>
the height, the said line. [57]

He asserts that this passage implies that ‘the base is equal to a given line, the height simple is a given line’ [59]. In fact, however, the practice of parallel constructions in Greek would incline most readers to supply the assertion about equality in the second phrase, given its occurrence in the first. This tendency is also felt in the English, although perhaps to a lesser degree. At any rate, this passage is not strong support for Netz’ case.

A common expression used by Greek mathematicians to assert a proportion is to claim that one ratio is the same ($\alpha\upsilon\tau\acute{o}\varsigma$) as another ratio.⁹ In his commentary to *SC* 1.13, Netz wants to argue for the possibility that Greek mathematicians felt that equal ratios could somehow be conceived as different from one another. I found his argument for this quite fantastic. In the course of *SC* 1.13, we encounter

ἔχει δὲ καὶ τὰ $KT\Delta, ZPA$ τρίγωνα πρὸς ἀλλήλα λόγον, ὃν αἱ
ἐκ τῶν κέντρων αὐτῶν δυνάμει,

which Netz translates as

but the triangles $KT\Delta, ZPA$ also have to each other <the>
ratio, which their radii <have> in square. [86]

In his commentary Netz remarks, ‘We find it very difficult not to attach the definite article to a well-specified ratio’ [90].¹⁰ On the basis of the absence of the definite article in the Greek, he argues that it is possible that Greek mathematicians considered the two ratios of a proportion as somehow different. In the first place, however, it is

⁹ I counted over 30 instances in *SC* alone.

¹⁰ Further on in this note, Netz claims that the concept of ratio ‘is not reducible to equalities and inequalities between numerical quantities’. This claim is based in large part on work by David Fowler [1987]. In fact, however, a close reading of Aristarchus’ *On the Sizes and Distances of the Sun and the Moon* shows that proportions and ratio inequalities were transformed into (and in this sense reduced to) equalities and inequalities. What is more, as Fowler apparently did not notice, the standard operations on ratios were sometimes performed directly on equalities and inequalities. See Fowler [1987, 246–248] for his discussion of Aristarchus.

risky to make claims about Greek conceptual habits on the sole basis of an omitted definite article, especially in a text that has undergone as much intervention as *SC*. In the second place, I fail to see how the definiteness, or indefiniteness, of the ratio in question has any bearing on how the Greeks conceived of proportionality. It is the relative clause that asserts the proportionality, not the article (or lack thereof). Whether *A* has to *B* a ratio which *C* has to *D*, or *the* ratio which *C* has to *D*, tells us nothing about how the objects of the relative clause are related to those in the primary clause. If we wish to know more about this relationship, we should look elsewhere in the mathematical corpus, for example, in the two surrounding sentences. In both of these we find proportions being asserted by claiming that one ratio is the same as another.

Mathematical Remarks

Reading Archimedes carefully is a difficult business, no matter what the language or presentation involved. In my experience, the greatest difficulty involved is that raised by the mathematical content itself. One often wants the aid of an overview to help elucidate the motivation for particular moves in Archimedes' argument. Because Netz provides no commentary devoted to mathematical discussion of this sort, many readers will find it necessary to refer to earlier treatments by Dijksterhuis [1987] and Heath [1912].

Netz himself provides three basic aids to following the details of the mathematics. (1) Generally following Heiberg, he includes footnotes that provide justification for specific steps in the argument by referring either to propositions that make up a 'tool-box' of elementary geometric knowledge or to earlier propositions in *SC*. (2) He tags passages in Archimedes with the page numbers in his translation of Eutocius where the *Commentary* explains a particular bit of mathematics. (3) He gives more general footnotes that are meant to clarify the line of argument.

On the whole, this apparatus is enough to elucidate Archimedes' approach, provided that the reader is familiar with ancient mathematical practices, has a good knowledge of Euclid, and the patience to work through everything from an ancient perspective. There are, however, a few places where I think Netz' remarks are off base.

Some of these are perhaps aesthetic, having to do with that elusive concept of mathematical elegance. For example, Archimedes asserts

$$\Gamma\Delta : MN = \Gamma\Delta^2 : H\Theta^2$$

directly on the basis of

$$\Gamma\Delta \times MN = H\Theta^2.$$

This follows because

$$\Gamma\Delta \times MN = H\Theta^2 \rightarrow \Gamma\Delta : H\Theta = H\Theta : MN,$$

of which the original assertion is simply a duplicate ratio. This kind of manipulation of ratios is quite common in Archimedes. Netz, however, makes a convoluted geometric argument based on objects that do not appear in the figure [188n14]. In other cases, Netz has simply not found the simplest justification.¹¹

Others have to do with misconstruing the mathematical prerequisites to the situation at hand. For example, Netz claims that the tangent EZ in *SC* 1.12 must be parallel to a certain diameter, given that a related line in *SC* 1.10 is so constructed; and he raises this issue for discussion in his commentary [81]. *SC* 1.10, however, is about a cone and triangles; whereas *SC* 1.12 is about a cylinder and parallelograms. The logic of *SC* 1.10 depends on EZ being parallel to the diameter. In *SC* 1.12, this condition is unnecessary; hence, EZ may be any one of the tangents between A and Γ . In a similar vein, Netz remarks in a note to *SC* 2.1 that Archimedes is wrong to assert that ‘each’ of the lines $\Gamma\Delta$ and EA are given [188n16]. $\Gamma\Delta$ and EA have been introduced as the diameter and height, respectively, of a cone or a cylinder which is given in volume. Netz claims that they are only given as a couple. The word ‘given’ in Greek mathematics, however, has a number of meanings, an important one of which is ‘arbitrary’. Since we are in the context of an analysis, here, Archimedes is quite right. One of the lines is given in the sense of ‘taken at the geometer’s discretion’ and then the other is given in

¹¹ For example, Netz’ argument at 206n119 involves four operations on ratios, when successive applications of separation and inversion will suffice. Archimedes often makes two operations in a single step; usually he notes this, but sometimes he does not.

the sense of ‘determined through geometric construction’. Eutocius shows how the one can be determined from the other [270 ff.].

In one place, Netz unfairly finds fault with Eutocius’ reasoning. In the course of *SC* 1.9, Archimedes compares two triangles that are not in the same plane and asserts that one is greater than the other. Netz evidently found the situation puzzling; he criticizes Eutocius, and includes a lengthy note adopted from Dijksterhuis which justifies Archimedes’ claim [64n69]. Eutocius’ lemma, however, is perfectly sound, given Greek standards. The procedure Eutocius follows is common in Greek geometrical works that treat solid geometry. In order to compare two figures in different planes, one must be constructed in the same plane as the other, effectively folding it into the receiving plane.

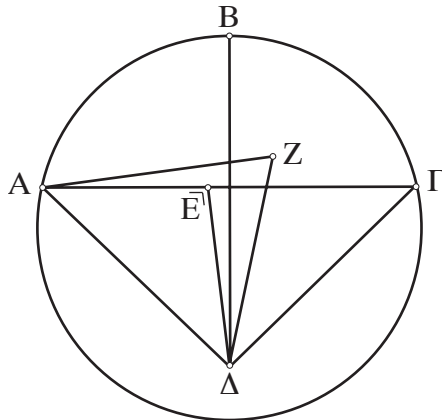


Figure 1. Eutocius’ diagram for *SC* 1.10

Circle $AB\Gamma$ is the base of a right cone and Δ is its vertex. $A\Gamma$ is a chord of circle $AB\Gamma$ and line ΔE is joined from the vertex perpendicular to $A\Gamma$.

Consider Figure 1 (adopted from Netz [257]). ΔE is drawn perpendicular to $A\Gamma$ so that $AE = E\Gamma$. As Eutocius shows, angle $A\Delta B >$ angle $A\Delta E$. Triangle $A\Delta B$ is folded down into the plane of triangle $A\Delta E$ by constructing angle $A\Delta Z =$ angle $A\Delta B$ in the plane of triangle $A\Delta E$. Line ΔZ is drawn equal to line $\Delta\Gamma$ and AZ is joined.

Eutocius simply asserts that $\text{triangle } A\Delta Z > \text{triangle } A\Delta E$.¹² The argument can be fleshed out a little. Since $\Delta Z = \Delta \Gamma$ and line ΔZ falls between ΔE and $\Delta \Gamma$, the point Z will lie somewhere beyond line ET . Hence, in the plane of $A\Gamma\Delta$, the triangle $AZ\Delta$ contains the triangle $AE\Delta$. Archimedes always displays a profound intuition for solid geometry and probably simply assumed this would be as obvious to the rest of us as it was to him.

Netz consistently follows Heiberg in justifying the operations of inversion and conversion by references to *Elem.* 5.7 cor., 5.19 cor., respectively. Heath [1956, 146, 174–175], however, has cogently shown that these corollaries do not result from the theorems that they follow. The corollaries were probably the work of a later editor who felt that *Elem.* 5 should provide an asserted justification for all the manipulations of ratios in general practice. The author of *Elem.* 5, however, may not have seen this as his project or may have considered these operations sufficiently grounded. Inversion follows as an almost immediate consequence of *Elem.* 5 def. 5, while conversion is simply successive applications of separation, inversion, and composition.

Final Remarks

This volume, and the overall project it launches, is a most welcome addition to scholarship on Hellenistic mathematics and the mathematical sciences. Its most successful contributions are probably the reassessment of the visual evidence as a fundamental source and the willingness to usher in entirely new ways of reading the ancient text. The translation will be useful for English readers who want a close approach to Archimedes' prose.

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¹² Netz [256n54] claims that this assertion is 'not necessarily true', despite the fact that he himself, following Dijksterhuis, proves a mathematically equivalent statement [64n69]. Eutocius' triangle $A\Delta E$ is the same object as Netz' triangle $A\Delta X$ [cf. 64, 256].

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