
Tablettes mathématiques de Nippur. 1er partie: Reconstitution du cursus scolaire. 2me partie: Édition des tablettes conservées au Musée Archéologique d'Istanbul, avec la collaboration de Veysel Donbaz et de Asuman Dönmez, translittération des textes lexicaux et littéraires par Antoine Cavigneaux by Christine Proust

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Reviewed by
Duncan J. Melville
St. Lawrence University
dmelville@stlawu.edu

The decipherment of cuneiform and the recovery of Mesopotamian mathematics, and especially that of the Old Babylonian period, opened an exciting early chapter in the history of mathematics. The pioneering work of Neugebauer, Thureau-Dangin, and others revealed a complex algorithmic and algebraic body of mathematics involving often ingenious constructions and presenting a sustained interest in quadratic mathematics more than 1000 years before Pythagoras. Inevitably, in this reconstruction, the fundamental problem texts documenting the extent and depth of the mathematical knowledge of the period took center stage. A key characteristic of Mesopotamian mathematics is that the sources are principally from an educational context: the tablets are school tablets produced by students and their teachers, and they derive from the business of teaching and learning mathematics. It is only comparatively recently that scholars have turned their attention from establishing the boundaries of Old Babylonian mathematical knowledge to determining how, and how much of, that mathematics was learned by students of the time. Proust's volume fits squarely into this current trend in understanding ancient pedagogy.

As archaeology and historiography have advanced in the past century, the value of an artifact's archaeological context has increased immeasurably. Regrettably, many of the key tablets whose contents provided the original reconstruction of Mesopotamian mathematics

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were purchased in the antiquities market for large university and museum collections during the 19th and early 20th centuries. Their archeological context is irretrievably lost and their origins may not be known to within 100s of miles and 100s of years. Christine Proust, however, is working with a collection of tablets from a well-defined locale (Nippur's scribal quarter) and time (mid-18th century BC), albeit a collection whose contents were considered sufficiently humble that they languished unread for a century after excavation. Between 1888 and 1900, John Peters and Hermann Hilprecht led the University of Pennsylvania's four Babylonian Expeditions to the city of Nippur, a city on the banks of the Euphrates that in the early second millennium had been famous as a religious and cultural capital renowned for its schools. In what became known as the 'scribal quarter', Hilprecht uncovered some 50,000 cuneiform tablets and fragments, of which some 800–900 had mathematical content. Under the rules of the Ottoman Empire, the finds were divided between the excavator and the state, with the result that about half of the mathematical tablets are in Philadelphia and a third are in Istanbul—the rest went to Jena with Hilprecht subsequently. Hilprecht published some 14 mathematical tablets in his reports on the excavations; the rest had to await later generations of scholars. Recently, Eleanor Robson [2001, 2002] has published the tablets from the University of Pennsylvania. Proust now presents over 300 previously unpublished tablets from the collection in Istanbul and uses the opportunity to give a detailed reassessment of the pedagogical production of mathematics in Old Babylonian Nippur.

The approach typically taken by Assyriologists to publishing collections of tablets is to present them in hand-drawn copies along with transliterations (rendering the cuneiform in modern script), translation, commentary, and a few photographic plates of some of the more important or difficult tablets. This format does not always sit well with publication of mathematical material, especially the kind of numerical tablets that form the bulk of Proust's collection. Proust follows the standard format for the lexical and literary tablets; but, for the rest of the mathematical tablets, she provides (very useful) composite tables in the text along with detailed catalogs, comparison of tablet contents, and excellent photographs of all the tablets on a CD that is included with the book. One must applaud this appropriate use of technology to deliver the maximum possible in-

formation into the hands of the reader while saving the author the labor of drawing hundreds of hand copies of metrological and numerical tablets, although I must note that my CD was damaged and only about half of the files were readable. However, it appears that the tablet images have also been made available through the Cuneiform Digital Library Initiative (CDLI) at <http://cdli.ucla.edu/>.

Apart from the edition of the Istanbul tablets, the bulk of the volume constitutes an overall analysis of mathematics at Nippur. The entire corpus can now be discussed as a whole for the first time since excavation—Proust also had access to the Jena tablets and incorporates them into the general account.¹ She opens with a chapter on sources that sets the scene, explains the location and importance of Nippur, and details the sequences of expeditions and the tablets excavated. Proust also describes the founding, growth, and development of the Istanbul collection and the place of the Nippur tablets within it. Some of Proust's analysis is statistical and she very carefully confronts the problems of selection bias in the sources. Beyond the accidents of archaeology, there also is the question of the choices made by the original scribes and students. In cases of preserved archives, one might expect that the finest, most important, or most necessary would be selected. The Nippur corpus in some ways presents the opposite picture, since most of the school tablets were either destined for recycling or had already been incorporated into walls and floors of later building-phases of the mud-brick houses. (Such recycling can lead to additional difficulties in reconstructing the original text.)

In chapter 2, 'Scribal Schools', Proust takes on the vexed question of where students learned, that is, of the 'schools' themselves, as well as outlining the overall course of study. We will take these points in reverse order. The Old Babylonian curriculum is reasonably well-understood, following the pioneering work of Veldhuis [1997] and Tinney [1998]. An analysis of catch-lines and different topics appearing on the same exercise tablet has helped establish a general chronology. The core of scribal education was learning to read, write, and speak Sumerian, at a time when this was a dead language and structurally unrelated to spoken Akkadian. This, not entirely practical, education fulfilled a desire for arcane knowledge and status among scribes

¹ Proust will publish the Jena tablets in a forthcoming volume with M. Krebernik.

and was largely accomplished by repeated copying of lists, from simple lists of syllables in the early phases to Sumerian proverbs and poems in the later phases. Mathematics was incorporated into this sequence beginning with repetition of metrological quantities and proceeding to arithmetic lists and culminating in computation. A central issue in the reconstruction of the Old Babylonian curriculum is determining what texts and lists were studied and in which order. There have been several attempts at such a reconstruction, most notably by Veldhuis [1997] and Robson [2001, 2002]. It is clear that there were certain common core texts and many other optional ones, and that the selection varied from place to place. Proust's detailed analysis of the Nippur corpus leads her to make the important and very plausible suggestion that, while there is an overall development from simple to complex texts, within each grouping students may have studied several different lists at one time, and that the search for a linear ordering is misguided.

On the question of the Old Babylonian school itself, the evidence is much less clear. Proust foregrounds the copying of Sumerian poems praising the scribal arts and extolling the importance of scribes, and argues that such a concern for generating an *esprit de corps* is better suited to an institutional framework than through family apprenticeship. On the other hand, the archaeology suggests that the schools were small, with perhaps one to five pupils, thus implying a more family-oriented approach. Andrew George [2005] has recently argued that the scribal 'school-days' literature may look back to a time of larger institutions in the preceding Ur III period.

In chapter 3, Proust turns to (abstract) numbers and metrological units. Old Babylonian metrology was a mixture of ancient systems, such as that for capacity,² and newer ones, such as those for weights and volumes. Many metrological units and relationships between them had been altered in the Sargonic and Ur III reforms of the later third millennium in order to create interconnecting systems that allowed easier computation, a bureaucratic convenience. Proust carefully explains the different sets of units for lengths, areas, volumes, capacities, and weights as well as the various notations for multiples

² Capacity units measure bulk foods and liquids, as opposed to volume measurements related to length units. Capacity relates to containers of various sizes, as we use cups, gallons and barrels, rather than cubic inches.

and standard fractions of the basic units. Her exposition, together with the tables in the appendix and the detailed treatment of metrological tables in the later chapters is probably the clearest and most comprehensive survey of Old Babylonian metrology yet written.

It is a truism among historians of Mesopotamian mathematics that the abstract sexagesimal place-value number system for which Old Babylonian mathematics is so well known is an artificial construct intended solely for calculation. As a floating-point system, it is hard to use for addition, although good for multiplication. Proust describes the basic notation of the standard system and reiterates her argument [2000] that part of the calculations of mathematical problems took place ‘off-tablet’. The additional evidence from the Nippur tablets strengthens her argument. In describing the artificial nature of the sexagesimal system, Proust argues that instead of numbers as such, they can be interpreted more profitably as numerical ‘strings’, given that multiplication tables are ordered lexicographically by left-most symbol.

In chapter 4, ‘Description of the Tablets’, Proust turns to the material culture of her topic, a theme of growing importance. The great mathematical text editions of the early and mid-20th century from Neugebauer to Bruins and Rutten emphasized texts in their titles and treatments: *Mathematische Keilschrifttexte* [Neugebauer 1935–1937], *Textes mathématiques babyloniens* [Thureau-Dangin 1938], *Mathematical Cuneiform Texts* [Neugebauer and Sachs 1945], *Textes mathématiques de Suse* [Bruins and Rutten 1961]. Proust has chosen to differentiate herself from her predecessors by her choice of title: *Tablettes mathématiques de Nippur*. While Neugebauer, especially, was careful to record the physical details of tablets, and introduced elements of a typology for table texts, commentary and analysis were principally text-based. In recent decades, a more detailed typological framework for analyzing scholastic tablets has been developed. Proust takes this standard analytical schema and uses it to extract considerable organizational information on a number of levels. Physically, the tablets are divided into six types. Proust shows how these types correlate with their content as well as how some tablets of a certain type (but not all) never mix categories of content. For example, the multi-column tablets with long extracts from metrological or mathematical lists or tables are always unified, whereas the daily tablets that feature copying on the obverse and rehearsal of previously learned material

on the reverse often do mix categories of content. Proust also notes that the so-called Type II tablets, which are very common at Nippur, are absent from the approximately contemporary corpus from Ur, although whether this is due to difference in pedagogy or accidents of survival and excavation one unfortunately cannot say. In this chapter, Proust also adumbrates the main division of her sources into metrological lists, metrological tables, and numerical tables, a division given detailed treatment in the following chapters. She observes that while these texts do follow clear organizational rules, one must be cautious in generalizing from a given collection as the rules vary from place to place.

Chapters 5 and 6, extending over 120 pages, present Proust's detailed reconstruction of mathematical education at Nippur. Chapter 5, which deals with elementary education, accounts for 90% of the Babylonian Expedition tablets. Mathematics in Old Babylonian Mesopotamia was still very concrete, concerned with computations involving physical objects expressed in series of everyday units. Thus, a pupil's first exposure to mathematics came in the form of memorizing long metrological series. Proust makes a distinction between metrological lists and metrological tables. Metrological lists give the sequence of quantities in a given metrological domain and provide practice in writing; metrological tables have the same list of quantities, but also convert them into sexagesimal multiples and fractions of a base unit, thereby training the student in writing the sexagesimal figures and in computation. Each series proceeds in increasing size from the smallest quantity up to some large unit; and the series were learned in the order of capacity, weight, area, and length. The first three apparently went by the names of grain, silver, and field, reflecting their origins; the length list does not seem to have had a name. Together, the four complete sequences run to some 620 entries, although individual tablets present extracts in a variety of sizes. The majority of exemplars are capacity lists and tables; lists and tables are never found intermingled on a single tablet. The base unit of length is the *nindan*, while the base unit of height is the *kuš* ($12 \text{ kuš} = 1 \text{ nindan}$). A few of the tables convert metrological lengths to sexagesimal heights and constitute the last metrological tables in the sequence to be studied.

After learning the written metrological notation and having practiced the conversion of metrological quantities into sexagesimals, the

pupil's next exposure to mathematics came in the form of purely numerical tables. The Nippur collection includes reciprocal (or inverse) tables, many multiplication tables, tables of squares, and tables of square and cube roots. In addition to providing much evidence for the standard view that multiplication tables were learned in descending order of principal number, Proust makes some other astute observations. The first concerns the tables of inverses. In Old Babylonian mathematics, division is achieved by 'multiplication by the reciprocal', or, more accurately given the floating point-nature of the sexagesimal system, 'multiplication by the inverse' as Proust prefers to call it. The Nippur inverse tables contain two entries at the beginning: 'of 1, its $\frac{2}{5}$ is 40; its half is 30'. Proust identifies these lines as a two-line table of fractions which she distinguishes from the remaining sets of inverses. Proust notes that this small table is an Old Babylonian innovation which does not appear in the Ur III examples.

Proust gives a very good analysis of the similarities and differences between the Nippur numerical tables, especially the multiplication tables, and those from other locations, thus illustrating the variability of the relatively standardized sources across Mesopotamia. She also argues that the tables of squares belongs with the multiplications as the last in the series. However, she sees the tables of roots not as giving the inverses of tables of squares, but as representing a new mathematical operation and tablet series that was sometimes introduced at the end of a student's study of mathematical tables.

The overall structure of the elementary level of mathematical education as reconstructed by Proust begins with extended practice in writing capacity lists, followed by shorter periods working on the remaining lists. Similarly, the tables begin with an extended period of capacities followed by briefer periods of the other metrological tables. Numerical tables begin with a short period on inverses, followed by a long time working through the multiplication tables and concluding with brief exposure to squares and roots. However, the interval spent working with capacity tables appears to overlap with later phases of metrological lists, and numerical tables make their appearance only slightly later than capacity tables. Proust discusses a number of possible interpretations of this juxtaposition, but retains a clear sense of the basic difference between lists and tables, suggesting that perhaps different students received different training. Certainly,

we should be cautious in ascribing too much homogeneity to scribal education, even education involving few students; and the fact that tables and lists never occur together is very striking. This is an important point of Proust's and deserves to be followed up in the study of other collections.

After the wealth of sources described in the chapter on the early phases of mathematical education, Proust has only some 40 tablets as witnesses to the more advanced stages. Most of these contain calculations of multiples or inverses. Proust describes the different ways of organizing multiplication, the numerical examples and applications which require finding the areas of squares and other quadrilaterals, as well as, on just three tablets, computing volumes. Within this context, Proust well illustrates her thesis of the disjunction between metrology and abstract computation. Among the assemblage is a group of tablets with exercises in computing areas of squares. The statement of the problem and its solution are written in metrological units in sentences in the lower right-hand corner of each of these tablets, while the multiplication involved is written in abstract numbers in the upper-left corner. This is as clear support as one could wish. Most of these area-computation tablets have been published previously and an image of the unpublished example adorns the cover of the volume.

Accompanying the exercises in multiplication is the problem of determining inverses of numbers not in the standard table. Proust notes that all such computations in the Nippur corpus use pairs derived from sequences of doubling and halving of one standard pair. The procedure used to pass from a number to its inverse has been described previously [see, e.g., Sachs 1947]. Proust relates the procedure and adds a very good section on how the physical layout of the numbers on the tablet acts as an aid to computation. Given the power of the method and its practical restriction to one sequence of numbers, Proust sees a tension between original creativity and a conservative pedagogy. As the sequences of doublings progresses, one soon achieves many-place numbers and these often have errors. Proust sees a pattern in these errors indicating that long numbers were divided at the fifth sexagesimal place, which she takes to imply a limitation on the size of the abacus (or similar 'off-tablet' computational device) used. Given the absence of direct archaeological evidence, I find her inference suggestive but not conclusive.

With respect to the few volume calculations, Proust notes that Old Babylonian metrology presents three different types of ‘volume’ units—area \times height, piles of bricks, and capacity measures—and that abstract volumes occur only in mathematical texts.

Out of the 800 mathematical texts under discussion, only three are problem texts with sequences of solved problems. All three have been published before: one by Hilprecht, the other two by Robson. Proust and Robson have found another fragment that joins one of the other tablets; that fragment is published here for the first time. All three texts contain problems concerning the calculation of volumes or of parameters derived from volumes. Proust re-publishes the tablets in full, discussing previous commentary and noting where her interpretation of these difficult and broken texts diverges from others. Additionally, she stresses the flow of conversion and computation, showing which tables support each step.

After this extensive and detailed survey of the Nippur corpus, Proust summarizes her results with commendable caution. One of her key findings is that education clearly varied from place to place and from pupil to pupil. Naturally, such variability makes generalization problematic. A few children trained as scribes. Students learned how to read and write metrological notation, with the curriculum dominated by the capacity series used (among other things) to measure grain; they learned by practicing with carefully structured lists. Some students learned how to reckon in the abstract sexagesimal system; they learned from structured tables, both metrological and numerical. Some students applied that knowledge in calculating areas or inverses. Abstract numbers were principally for multiplication and finding inverses. There is not much evidence for mathematical problem-solving beyond the computations of areas, for reasons that are unclear.

How far can these results be generalized? While details vary from location to location, the key themes of writing, structured lists, and tables, and the restriction of the abstract numeration system to computation are universal across Old Babylonian Mesopotamia. The final publication of the Istanbul tablets after a century of neglect is a noteworthy event. By going beyond mere publication of the tablets and providing a synoptic view of mathematical education, Christine Proust has produced an invaluable volume. Her clear and carefully

detailed exposition, her concern for both text and tablet, and her extensive statistical analysis (summarized in the text but presented fully in appendices) show that she has mastered modern historiographic techniques. The result is up-to-date and comprehensive.

The fortuitous use of clay as a writing medium means that Mesopotamian scribes have left a legacy unique among ancient cultures whereby historians can reconstruct in detail both the content of education and its pedagogy. There is much in this book for experts; but there is also a great deal for readers from outside the field who have an interest in education, pedagogy, and schooling.

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