Islamic Astronomical Tables: Mathematical Analysis and Historical Investigation by Benno van Dalen

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Reviewed by
José Chabás
Universitat Pompeu Fabra, Barcelona
jose.chabas@upf.edu

For the last 25 years, Benno van Dalen has devoted his efforts indefatigably to the study and analysis of medieval astronomical tables, particularly Islamic tables, and his research has contributed substantially to the progress in this field since the pioneering works by O. Neugebauer and E. S. Kennedy, among others. Van Dalen's writings have appeared in various books, and journals; these are not always easy to obtain and it is most welcome that nine of his most remarkable papers, dating from 1989 to 2008, are now published together in a single volume. The papers presented here are grouped into two categories, one concerning methods for analyzing astronomical tables (five chapters) and the other devoted to the study of various *zijes*, that is, astronomical handbooks with tables and explanations for their use (four chapters).

The core idea underlying the analysis of astronomical tables is that the parameters embedded or explicit in them are key indicators of their dependence on previous tables compiled by previous authors and thus provide a secure way to unveil the transmission of astronomical knowledge. More specifically, van Dalen has focused on the development of statistical estimators for the parameters in astronomical tables, such as the LNEC (Least Number of Errors Criterion), a mathematical criterion to determine the range of values of a parameter containing the largest number of values 'correctly computed' [see VI.7]. Of course, the crucial issue here is how to decide what a correctly computed value is. For this, van Dalen uses the concept of tabular error, defined here as the difference between an entry in a table and the corresponding value derived from the function underlying the table [see II.144]. In the cases examined in this volume, statistical analysis of tabular errors proves to be a powerful tool to 'crack' a table and to put it into context; but it has to be handled carefully to avoid potential anachronisms. To be sure, the concept

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of a function as understood nowadays does not necessarily apply to all the material that the medieval table-maker had in front of him to compile an astronomical table. This, together with computational methods that are not always consistent with those in use at the present time, can lead to values perfectly computed but slightly differing from those derived from a nice modern function.

In chapter 1, van Dalen uses statistical methods to obtain approximations to a parameter in an astronomical table and introduces statistical estimators in order to establish confidence intervals with a fixed probability of containing the unknown parameter. He focuses on two cases: the determination of the value of the obliquity of the ecliptic in a right ascension table and the value of the solar eccentricity in a solar equation table. In both cases, we are faced with a single unknown parameter. Van Dalen discusses the use of both a weighted estimator and the maximum likelihood estimator to determine its value. The results are then compared with the attested values for these parameters in the astronomical literature. The appendices provide further details of these estimators as well as a clarification of the concept of tabular error, of which three categories are mentioned: scribal errors, computational errors, and rounding errors.

In *Almagest* 3.9, Ptolemy defines the equation of time but does not tabulate it. This he does in the *Handy Tables*, using the true solar position as the independent variable and giving entries to minutes and seconds of an hour at intervals of one degree. Van Dalen analyzes this table in chapter 2, originally published in 1994. The purpose is to establish its underlying parameters and to explain the computational methods used by the author. Inspection of the successive tabular differences already leads van Dalen to conclude that Ptolemy used linear interpolation within intervals of 6°, giving rise to a distributed linear interpolation in the full table. Estimation of the parameters by the least squares method confirms that the independent variable is the true, not mean, solar longitude and, quite successfully, assigns the embedded parameters (obliquity of the ecliptic, solar eccentricity, longitude of the solar apogee, and epoch constant) 95% confidence intervals around historically plausible values. As for the methods of computation, van Dalen concludes that Ptolemy used a rounded value of 66° for the longitude of the solar apogee instead of the attested value 65;30° and linear interpolation between right ascension values for every 10°. These results were confirmed by Raymond

Mercier in a later analysis, although with a different approach, of the same table [2011, 103].

Next, the author analyzes a table to compute the true solar longitude extant in a manuscript containing the *Jāmiʿ zij* by Kūshyār ibn Labbān (*ca* AD 964). In chapter 3, van Dalen uses two methods: the least squares estimation, already applied in chapter 2, and a rather sophisticated method involving Fourier analysis and the development of a 360°-periodic function as a Fourier series. The good results obtained for the parameters underlying the table point towards Yaḥyā ibn Abī Manṣūr (*ca* AD 830) as the author of this table.

The equation of time is again addressed in chapter 4, in the framework of a general study of the astronomical tables in al-Khwārizmī's *Sindhind zij* in the version by Maslama (*ca* AD 980), the only one extant. Application of the method of the least squares makes it possible to determine the structure of the table and the values of its embedded parameters: an obliquity of the ecliptic of 23;51° (a rounding of Ptolemy's value, 23;51,20°), a factor of 15°/h for the conversion from hours to time-degrees, a maximum solar equation of 2;14° (thus different from Ptolemy's), and a longitude of the solar apogee of 82;39° (the value used in the earliest Islamic *zijes*, among others). As was the case with Ptolemy's equation of time, the independent variable is found to be the true solar longitude.

In chapter 5, van Dalen applies the mathematical technique LNEC to the analysis of Rajah Jai Singh's tables for the mean motion tables of the Sun, the Moon, and the five planets, completed around 1735 in Jaipur, India. The author convincingly shows that they derive from Philippe de la Hire's *Tabulae astronomicae* (Paris, 1702 and 1727). Indeed, a copy of these tables was brought to Jaipur in 1730 by the Jesuit missionaries at the Rajah's request.

The *Nāṣirī zij* by Maḥmūd ibn ʿUmar, the earliest *zij* written in India, is the subject of chapter 6. This voluminous *zij* was compiled in the middle of the 13th century; only the tables for determining planetary longitudes are addressed here. The application of the LNEC to the tables for the mean motions provides good results, for it shows agreement with the Byzantine version by Gregory Chioniades (*ca* AD 1305) of the lost *ʿAlāʾ ī zij* by al-Fahhād al-Shirwānī (*ca* AD 1180), indicating that this was the source of the *Nāṣirī zij*. The tables for the planetary equations are basically of the standard type and in all of them we find displacements of 12 zodiacal signs to avoid the use of subtractive values, a feature not uncommon in Arabic *zijes*. In this

chapter, the author outlines a project of his to compile a list of all known Arabic *zijes* (now up to 250) with basic information on them (author, title, date, geographical origin, available manuscripts, and so forth).

In chapter 7, van Dalen provides a detailed description of a manuscript now at the University Library of Leipzig, MS Voller 821. He was thus afforded the opportunity to find materials therein from the early period of Islamic astronomy, including a second copy of the *Mumtaḥan zīj* by Yaḥyā ibn Abī Manṣur (*ca* AD 828), together with chapters and extracts from works by Ḥabash al-Ḥāsib (*ca* AD 860), al-Battānī (*ca* AD 900), and Ibn al-A'lam (10th century). The only copy previously known is at the library of El Escorial (MS Árabe 927). The Leipzig manuscript that van Dalen examined was probably copied in northern Iraq around 1200 and was bound in great disorder, so that part of van Dalen's work has consisted not only in identifying the authorship of the various tables and texts but in rearranging the manuscript.

The purpose of chapter 8, written in collaboration with F. S. Pedersen, is to make an inventory of the problems related to the transcription of al-Battānī's *zij* with a view to a possible new edition of it. In this chapter, the authors focus on a re-edition of the table for the solar declination (obliquity = 23;35°) and display the apparatuses, which are not easy to read, for a variety of tables in this *zij* (sine, cotangent, half excess of the longest daylight, mean motion of the northern lunar node, lunar equation of anomaly, and lunar latitude).

Chapter 9, originally written as an entry in the *Encyclopaedia of Islam* (2000), deals with chronology and presents valuable information on the various calendars and eras used in the Islamic World. Most useful are Tables 1 and 2, which contain precise information on the calendars used in a dozen *zijes* from the ninth to the 15th centuries, and the most common epochs (Hijra, Alexander, and Yazdigird, among others) used in them, together with the differences in days (both in decimal and sexagesimal notation) between these epochs. We are also given formulas and examples, without appealing to Julian day numbers, for counting the number of days from one epoch to a given date and, conversely, for computing the number of years between a given date and a given epoch.

Indices of subjects, personal names, titles, localities, and manuscripts complete a valuable volume offering only a part of the fruitful work produced by Benno van Dalen in the last 25 years. It is to be hoped that he will keep generating more and more results of his high-level research on mathematical

astronomy in Islam, creating new tools for table analysis and providing a long awaited updated version of Kennedy's *A Survey of Islamic Astronomical Tables*, originally published in 1956.

BIBLIOGRAPHY

Mercier, R. 2011. *Ptolemy's Handy Tables*. vol. 1b. Louvain-La-Neuve, BE.