
Conformément aux observations d'Hipparque. Le papyrus Fouad inv. 267 A
by Jean-Luc Fournet and Anne Tihon with an Annex by Raymond Mercier

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This is not a normal book review in several ways. First, I am not attempting to review the entire book, of which part 1 consists of an edition, translation, and notes on Papyrus Fouad 267A by Jean-Luc Fournet and Anne Tihon, followed in part 2 by a lengthy commentary by Tihon on the astronomical aspects of the text. Rather, I am reviewing only the annex, which consists of tables and a summary analysis by Raymond Mercier. Second, since in the opening paragraph of his summary analysis Mercier points out that 'there is room for a concise analysis of the model, as seen from a more purely mathematical perspective', my review will of necessity have more mathematical and technical content than a normal book review. Third, my own analysis depends heavily on the analysis and reconstruction of the underlying tables in the papyrus by Alexander Jones that was circulated in 2009 and published in 2010a, and on a preliminary analysis of the solar model underlying the papyrus by John Britton that was circulated in 2009, both based on the report by Tihon on the investigation of the papyrus at a conference in 2007 that was published in 2010.

P. Fouad 267A appears to be a worked example of the calculation of the Sun's position for a date in AD 130. Two main parts of the papyrus are preserved. The first part, on the recto, gives the intermediate and final results of what is apparently a calculation from tables of the increment in mean solar longitude using three different year lengths, corresponding to tropical, sidereal, and mean (what we call Julian) years. The results are fragmented but nevertheless are complete enough to allow a full reconstruction of the mathematical basis underlying the tables [Jones 2010a, 41n46]. The second part, on the verso, is even more fragmented and gives the final tropical and sidereal solar longitudes as well as a calculation from a table of ascensions

of the declination of the Sun and the length of seasonal hours on the date of the example. In particular, nothing survives regarding the computation of the equation of center connecting the mean and true longitudes. Hipparchus is prominently mentioned several times in the first part, once regarding a sidereal year length and once regarding observations related to a summer solstice in -157 .

Mercier's mathematical analysis is, as I will show below, quite unconventional. I will, therefore, first present a standard and fully conventional analysis, primarily to see if it works. The conventional eccentric solar model is shown in Figure 1, p. 92 below. The Sun at S moves around the ecliptic, a circle of radius R and center C, at a constant speed as seen from C. The Earth is at O, a distance e from C, so the apparent speed of the Sun is slowest when the Sun is at the apogee A and fastest when at the perigee P. The mean anomaly α is $\angle ACS$ and the true anomaly κ is $\angle AOS$. The equation of center q is $\angle CSO$ and by convention $\alpha + q = \kappa$, so q is negative when $\alpha < 180^\circ$ and positive when $\alpha > 180^\circ$. The Earth-Sun distance $\rho = OS$ is determined by

$$\rho^2 = (R + e \cos \alpha)^2 + (e \sin \alpha)^2$$

and, by the law of sines applied to $\triangle CSO$, we have

$$\sin q = -(e/\rho) \sin \alpha = -(e/R) \sin \kappa.$$

The papyrus distinguishes three frames of reference for the solar motion. One is based on a year of very nearly $365\frac{1}{4}$ days and plays no role in the following. The frames that do play a role are the sidereal and tropical frames. In the sidereal frame, longitudes are measured from a point fixed relative to the background stars, while in the tropical frame the longitudes are measured from the vernal equinoctial point determined by the intersection of the ecliptic with the celestial equator. The sidereal speed ω_s of the Sun in the papyrus is determined from the period relation $37,473\frac{1}{3}$ revolutions in 37,500 Egyptian years of 365 days, and the tropical speed ω_t is determined from the relation $37,474\frac{2}{3}$ revolutions in 37,500 years. The difference in these speeds, $\omega_\pi = \omega_t - \omega_s$, is due to precession and is 8° in 625 years, or 1° in $78\frac{1}{8}$ years [Jones 2010a, 29–30, 43n46].

In Figure 1, the directions of the sidereal and tropical zero-points are shown. As a consequence of precession, the tropical zero-point will rotate relative to the sidereal zero-point in the clockwise direction with speed ω_π . Relative to these directions, the mean and true longitudes of the Sun are, for a longitude

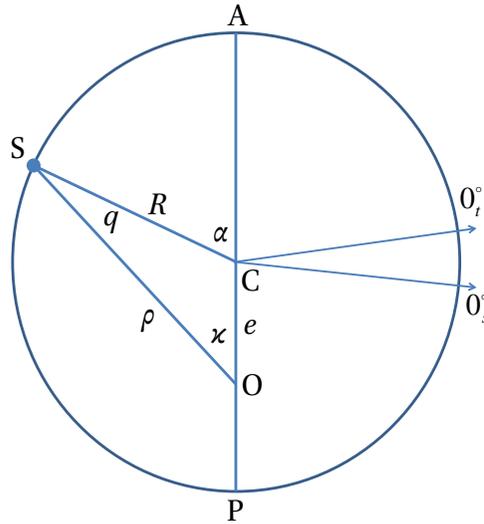


Figure 1. Conventional eccentric solar model

of apogee A , $L = \alpha + A$ and $\lambda = \kappa + A$, and so $q = \lambda - L$. Note that, for any moment in time, the numerical values of the mean and true longitudes of the Sun and the longitude of the apogee depend on the directions of these zero-points, but that the angles of mean anomaly α , true anomaly κ , and the equation of center q are independent of the frame of reference.

The papyrus computes an example for a date $T_3 = +130$ Nov 9 at 3 am or JD 1768852.625 (all dates are relative to Alexandria). The author, presumably using tables based on the period relations given above, computes the change in three mean longitudes by summing the changes in 37,788 Egyptian years of 365 days, three 30-day months, 19 days, and 21 hours. Thus, the ‘ancient’ epoch T_0 of the tables was 13,792,729.875 days earlier on $-37,632$ Jun 2 at 6 am or JD $-12,023,877.25$. It will be useful to consider also a ‘modern’ epoch T_1 37,500 Egyptian years after T_0 , which is -158 Oct 2 at 6 am or JD 1,663,622.75, and a date T_2 for a summer solstice associated with Hipparchus which is -157 Jun 26 at some ‘hour of day’, meaning during daylight, with the numeral of the hour unfortunately missing on the papyrus.

The two sums that we need have been reconstructed [Jones 2010a, 29–30, 43n46] and are the sidereal increment in mean longitude with a value of

154; 33, 53° and the corresponding tropical value 278; 15, 18°. The mean anomaly of the true Sun on the date T_3 is about 156; 15°, so we assume that 154; 33, 53° is also the value of the solar mean anomaly α . It then follows that the mean anomaly was zero at time T_0 and that the solar apogee is sidereally fixed [Britton 2009]. We further assume that the solar motion is eccentric and that e/R has the Hipparchan value $2; 30/60 = 1/24$. Then, the equation of center

$$q = \arcsin(-(e/\rho) \sin \alpha) = -1; 3; 55^\circ,$$

which is close to the equation of center of the real Sun (about $-0; 56, 43^\circ$) and, hence, $\kappa = \alpha + q = 153; 29, 58^\circ$. The papyrus gives the true sidereal longitude of the Sun as $\lambda_s = 228; 29, 44^\circ$; so the longitude of the apogee in the sidereal frame is

$$A_s = \lambda_s - \kappa = 74; 59, 46^\circ$$

and the mean sidereal longitude is

$$L_s = \lambda_s - q = 229; 33, 39^\circ.$$

The papyrus also gives the true tropical longitude of the Sun as $\lambda_t = 224; 20, 18^\circ$. Thus, we find

$$A_t = \lambda_t - \kappa = 70; 50, 20^\circ \text{ and}$$

$$L_t = \alpha + A_t = 225; 24, 13^\circ.$$

Note that by using the frame independence of α , κ and q we have been able to deduce the tropical values without ever using the tropical value $278 : 15, 18^\circ$ computed using the tables. The difference in longitude of the sidereal and tropical zero points is

$$\lambda_s - \lambda_t = L_s - L_t = A_s - A_t = 4; 9, 26^\circ.$$

The mean sidereal longitude at the ancient epoch T_0 is

$$L_s(0) = 229; 33, 39^\circ - 154; 33, 53^\circ = 74; 59, 46^\circ = A_s = A_s(0)$$

and so $\alpha = 0$ at T_0 , as assumed. Since the increment in tropical mean longitude from the ancient epoch T_0 to T_3 is $278; 15, 18^\circ$, and since the increase in precession is $123; 41, 25$, the tropical mean longitude at T_0 was

$$L_t(0) = L_t - 278; 15, 18^\circ = 307; 36, 56^\circ$$

and the tropical apogee at T_0 was

$$A_t(0) = A_t - 123; 41, 25^\circ = 307; 36, 56^\circ.$$

Once again, then, $\alpha = 0$, providing a consistency check on the entire reconstruction. The longitudinal difference of the sidereal and tropical zero points is

$$L_s(0) - L_t(0) = 127; 50, 51^\circ.$$

Similarly, at the modern epoch T_1 , we find

$$L_s(1) \& = 194; 59, 46^\circ,$$

$$L_t(1) \& = 187; 8, 55^\circ, \text{ and}$$

$$A_t(1) \& = 67; 8, 55^\circ.$$

So $\alpha = 120^\circ$ and $L_s(1) - L_t(1) = 7; 50, 51^\circ$.

At the modern epoch $T_1 = -158$ Oct 2 at 6 am the increment in mean longitude since T_0 and, hence, the value of the solar mean anomaly, is exactly 120° . The increment in precession is also 120° ; so the increment in tropical mean longitude is exactly 240° . The tropical longitude of the bright star Regulus, which was often used as a reference star in antiquity, was very near to 120° at this time. Perhaps these facts are more than coincidences and played a role in the foundation of the solar model; but if so, the details of the connection remain obscure, at least to me. It is also the case that exactly five days prior to T_1 , hence, on -158 Sep 27 at 6 am, Hipparchus reported an autumn equinox according to Ptolemy's account in the *Almagest* [Toomer 1984, 133]; and indeed the conventional model we are discussing as well as Mercier's model discussed below agrees very closely with the report of Hipparchus. Mercier suggests that the foundation of the solar model might have been somehow connected to that event but once again the connection remains obscure.

The papyrus gives the date but not the hour of the summer solstice in -157 ; so we have to pick the hour that results in $\lambda_t = 90^\circ$. That hour is about 9 pm; so $T_2 = 1, 663, 890.375$. This hour conflicts with the papyrus phrase 'hour of day' which seems to suggest that the solstice occurred before sunset.

Mercier's analysis begins with a discussion of the tabulated sums in the papyrus. He gives the period relation underlying the speed in precession, ω_π but for the sidereal and tropical speeds he gives only the numerical values $\omega_s = 0.9856$ and $\omega_t = 0.985635068493$, both in units of degrees per day. Both numbers are correct but it would surely have been more informative to give the underlying period relations, which are simple rational fractions. Next, Mercier assumes that the sidereal quantity $154; 33, 52^\circ$ is the mean

anomaly of the sidereal Sun and then, assuming an eccentric model, he finds a sequence of (E, A_s) pairs, with $E = R/e$, that are solutions to the equation

$$\alpha + A_s = \lambda_s + \sin^{-1}(\sin(\lambda_s - A_s)/E),$$

which is simply the equation $\alpha + q = \kappa$ given above. Mercier solves his version of the equation by iteration, even though it is simple to solve directly as shown above. For values of E between 22 and 26, and so for values of e between 2.31 and 2.73, the only resulting value of A_s that is near an integer, namely, $A_s = 74.997^\circ = 74; 59, 49^\circ$, is paired with the Hipparchan value $e/R = 1/24$. He then concludes that the sidereal apogee must be exactly 75° , so that the mean longitude is

$$L_s = \alpha + 75^\circ = 229; 33, 52^\circ.$$

Turning to the tropical quantity 278; 15, 18° , Mercier recognizes that this is far too large to be a mean anomaly if the true longitude is the papyrus value $\lambda_t = 224; 20, 18^\circ$; so he subtracts 120° from it and gets a value 158; 15, 18° , which he then treats as a mean anomaly α' in the tropical frame. He then proceeds to solve

$$\alpha' + A_t = \lambda_t + \sin^{-1}(\sin(\lambda_t - A_t)/E).$$

Once again he finds that for $22 < E < 26$, the only value of A_t close to an integer is

$$A_t = 67.003^\circ = 67; 0, 11^\circ.$$

Since this A_t is also paired with $E = 24$, he concludes that the tropical apogee must be exactly 67° , so that the mean tropical longitude is

$$L_t = \alpha' + 67^\circ = 225; 15, 18^\circ.$$

At this point Mercier has departed far from any conventional solar model. He has assumed that, at the same moment in time, $T_3 = +130$ Nov 9 at 3 am, the Sun has two mean anomalies, 154; 33, 52° and 158; 15, 18° and, hence, two values of the equation of center q , $-1; 4, 8^\circ$ and $-0; 55, 0^\circ$, and true anomaly κ , 153; 29, 44° and 157; 20, 18° . He also assumes that both apogees, $A_s = 75^\circ$ and $A_t = 67^\circ$, are fixed in their respective frames for all time. Since those frames move with respect to each other with the speed ω_π , the Sun in this scheme will in general have two distinct apogees. For example, at the time T_3 of the example, the sidereal apogee A_s is 75° and the zero-points of the sidereal and tropical frames are, in Mercier's scheme, $L_s - L_t = 4; 18, 34^\circ$ apart; so the tropical apogee A_t is, relative to the sidereal zero-point, at

$$67^\circ + 4; 18, 34^\circ = 71; 18, 34^\circ.$$

But since the direction of the apogee is a unique direction in space that all observers would agree upon, namely, the direction in which the Sun has the slowest angular speed, this is a physically impossible situation.

In any event, Mercier's relations for the time dependence of the mean longitudes are

$$\begin{aligned} L_s(t)\& &= 229; 33, 52^\circ + \omega_s(t - T_3), \\ L_t(t)\& &= 225; 15, 18^\circ + \omega_t(t - T_3), \text{ and} \\ L_{\pi}(t)\& &= L_t(t) - L_s(t) \\ \& &= 4; 18, 34^\circ - \omega_{\pi}(t - T_3). \end{aligned}$$

However, when these equations are used to compute the true longitudes at T_3 , one finds

$$\begin{aligned} \lambda'_s &= 228; 29, 57^\circ \text{ and} \\ \lambda'_t &= 224; 20, 6^\circ, \end{aligned}$$

which do not match the papyrus values $228; 29, 44^\circ$ and $224; 20, 18^\circ$. Since the papyrus gives the mean and true longitudes to two fractional places of precision, this sort of discrepancy must be expected from the rounding of the apogees, which by Mercier's calculation differ from integers in the second fractional place by about $0; 0, 11^\circ$. However, at the time T_0 of the ancient epoch, Mercier's relations yield

$$\begin{aligned} L_s(0)\& &= 75^\circ, \\ L_t(0)\& &= 307^\circ, \text{ and} \\ L_{\pi}(0)\& &= 128^\circ; \end{aligned}$$

and at the time T_1 of the modern epoch,

$$\begin{aligned} L_s(1)\& &= 195^\circ \\ \& &= L_s(0) + 120^\circ, \\ L_t(1)\& &= 187^\circ \\ \& &= L_t(0) + 240^\circ, \text{ and} \\ L_{\pi}(1)\& &= 8^\circ \\ \& &= L_{\pi}(0) - 120^\circ \end{aligned}$$

All of these integer values at the two epochs are certainly more pleasing than the epoch values found above using the conventional solar model and

assuming that the papyrus values for the true longitudes have been correctly computed.

Thus, the question arises: Can something be changed in the conventional analysis so that we recover these same integer values? The answer to that question is 'Yes', if we assume some mistakes on the part of the person who computed, in the lost lines between the recto and verso, the true longitudes in the papyrus. First, the writer computed the sidereal longitude nearly correctly. At time T_3 , he would get the mean anomaly $\alpha = 154; 33, 53$ but the equation of center $q = -1; 4, 9^\circ$ instead of the correct $-1; 3, 55^\circ$ and, hence, the true longitude $\lambda_s = 228; 29, 44^\circ$ instead of the correct $228; 29, 58^\circ$. Such an error in the second fractional place of the equation q is hardly surprising and could arise from any number of ways during the relatively complicated computation of

$$q(\alpha) = \arcsin\left(\frac{-e \sin \alpha}{\sqrt{(R + e \cos \alpha)^2 + (e \sin \alpha)^2}}\right)$$

or it might be that the writer was correct but that his tables for the equation of center to two fractional places, unprecedented in antiquity as far as we know, were faulty in the seconds place.

At this point, the writer could get the true tropical longitude by simply subtracting from λ_s the effect of precession at time T_3 , which is given by $128^\circ - 123; 41, 25^\circ = 4; 18, 35^\circ$, so his computed true tropical longitude would be $\lambda_t = 224; 11, 11^\circ$ instead of the correct $224; 11, 23^\circ$. However, it seems our writer instead took the longer route of computing the tropical longitude from first principles. Knowing the increment in mean longitude at time T_3 from his tables as $278; 15, 18$, he should have computed the apogee at the same time from precession as $A_t(3) = 307^\circ + 123; 41, 25^\circ = 70; 41, 25^\circ$. But it seems that here he makes a major mistake, adding only 120° , the effect of precession from T_0 to T_1 , but omitting the precession effect $+3; 41, 25$ from T_1 to T_3 . Thus, he got $\alpha = 158; 15, 18^\circ$ instead of the correct $154; 33, 53^\circ$ and using this value for α he got $q = -0; 55, 0$ instead of the correct (for the wrong α) $-0; 55, 12$. So his final true tropical longitude is $\lambda_t = 224; 20, 18^\circ$ when it should be $224; 11, 23^\circ$.

Of course the writer—perhaps he was a student—should have realized that he was making errors when he got different values for α and then for q in the sidereal and tropical frames. Maybe that explains why he, or perhaps

more likely, his teacher tore the papyrus in half and threw it into the trash bin where it was found many centuries later.

In conclusion, Papyrus Fouad 267A introduces us to a new solar model from antiquity similar to, but differing in many details from, the solar model of Ptolemy's *Almagest*. Unfortunately, the summary mathematical analysis provided by Raymond Mercier in the annex is severely flawed. Mercier proposes a mathematical model that has several properties that are completely unphysical: mean anomaly α , true anomaly κ , and equation q have different values in different reference frames; the apsidal lines of the solar orbit point in different directions in different reference frames; and the proposed equations for the time dependence of the mean longitudes do not reproduce the actual values from which those equations were determined. Strangely enough, Mercier makes no attempt to explain these utterly unphysical features; indeed, he does not even acknowledge them. Perhaps, if the underlying mean and true longitudes found in the papyrus were really inconsistent, such departures from convention could be justified. But as shown above, all the data in the papyrus are easily explained assuming conventional ideas well known in antiquity. Therefore, while there are many reasons to commend the book by Fournet and Tihon, the contents of the annex are not among them.

After this review was submitted, two additional papers related to P.Fouad 267A appeared. First, Jones 2016 contains both an English translation of the Greek text and a very extensive analysis of the astronomy found on both the recto and the verso of the document, and discusses how it fits into the context of what is known from many other solar models from antiquity. Second, Tihon and Fournet 2016 translates and discusses the contents of a small fragment now understood to be the upper part of the verso of P.Fouad 267A. The paleography of the new fragment appears to date the fragment and, hence, P.Fouad 267A to the third century AD instead of the second century tentatively suggested by the date AD 130 of the mean motions.

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