
Jacopo da Firenze's Tractatus Algorithmi and Early Italian Abacus Culture edited by Jens Høyrup

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The Italian Renaissance is remembered best for its magnificent works of art and architecture. The words themselves evoke the paintings and sculptures of Michelangelo, Leonardo, Raffaello, Botticelli, and the other great masters of the period, while the churches, palaces, and plazas of Italy continue to inspire wonder to this very day. The Renaissance is also remembered for certain masterpieces of modern literature and philosophy such as the essays of Petrarch, the stories of Boccaccio, and *The Prince* of Machiavelli; but the other accomplishments of the period are largely forgotten.

One of those achievements that is least remembered is the Renaissance contribution to mathematics. Although only a few experts are aware of it, the Italian Renaissance created the style and manner of doing mathematics that has become the common heritage of Western Europe and modern world culture. The mathematicians of the period, although virtually unknown by name, determined the way in which we write and calculate with numbers, the types of problems we solve, the manner in which we approach them, and, most significantly, the way we do algebra [see [Van Egmond 1986](#)].

The records of these achievements are preserved in a large body of documents known collectively as the ‘*abbaci*’, a name derived not from the more familiar reckoning device (the abacus, written with one ‘b’) but from the title of the fundamental work of the genre, the *Liber abbaci* of Leonardo Pisano or Fibonacci, which was composed in Italy in 1202.¹ More than 400 such documents survive from the

¹ There is now a complete English translation by Laurence E. Sigler [[2002](#)], which was reviewed by Serafina Cuomo in *Aestimatio* [[2004](#)].

broad Renaissance period itself, from about 1300 to 1600, in both printed and manuscript forms.²

Unfortunately, the elementary and inelegant form of these simple mathematical texts has kept them from receiving the kind of editions and printings that would attract the attention of other Renaissance scholars, let alone the general public. Those editions of the *abbaci* that do exist are largely typescript transcriptions cheaply reproduced and distributed by academic centers and small publishers.³

The current volume, an edition of Jacopo da Firenze's *Tractatus algorismi* prepared by Jens Høyrup of Roskilde University in Denmark, is the first to aim at a larger audience. It is the first text of its kind to be published by a major academic publisher, and the first to be translated into English in its entirety. This makes it the only example of an *abbacus* book that will be readily available to the wider academic and general public, and on this score alone it is worthy of some note.

The quality of the production is excellent. The typography is clear and readable, the paper and binding are of the highest quality, and all the drawings and diagrams have been redrawn to increase their readability. The English translation is printed in parallel columns so that it can be easily compared with the original Italian. The editor's deliberate decision to render a highly literal translation means that the English reads somewhat awkwardly at times; but since we are dealing with a technical mathematical text where content is more important than literary style, this is of little consequence.

The text itself is also well chosen. It provides a representative sample of what a normal *abbacus* book contained, including sections on how to write and calculate with the Arabic numerals, an exposition of the principle of place value, a large number of multiplication and division tables, and many practical business problems on pricing, exchange, interest and discount, partnerships, and the like, plus additional sections on practical geometry, algebra, and the alloying of

² For a nearly complete catalog of almost all known *abbacus* books and manuscripts, see [Van Egmond 1980](#).

³ See, e.g., the long series of 'Quaderni' published by the Centro Studi della Matematica Medioevale at the University of Siena, or the many editions of Gino Arrighi listed in the bibliography of the book under review.

metals. There is also a short list of the common coins in use at the time and their values in terms of their precious metal content. Thus, anyone who is interested in seeing what a typical *abbacus* book looked like can now readily refer to this volume.

The transcription is accurate though not perfect. I have checked the entire text against the original manuscript and found, on average, about one error per manuscript page. These are all confined to single letters or words and none of them affect the meaning of the text. The most annoying error is a systematic rendering of the numeral '1' as the lowercase letter 'j', a mistake so obvious and so persistent that I can only attribute it to some uncorrected computer glitch. The only error that even slightly affects the sense of the text is the rendition of the word 'terza' in the explanation of the rule of three on page 237 as 'altra'. This loses the significance of using the 'third' number that gives the rule its name.

The original manuscript is full of colorful drawings of buildings, objects, and people that illustrate many of the problems being posed. While these are quite crude and of no artistic merit, they add much to the charm of the original document. Unfortunately, apparently for technical and financial reasons, these were not reproduced but were all redrawn by hand by the editor and reproduced as black and white photographs. Again, these are quite accurate and do not affect the content of the text. However, I did note a failure to reproduce the numeral forms on page 196 accurately and to space the tables of continued division on pages 221–226 correctly. The former gives the wrong antique forms of the numerals 3 and 4 and omits a 1 written before the zero. This, when combined with the reformatting of the tables, might give the impression that the author wrote the zero separately and not always as part of the number 10.

These minor errors do not detract from the value of having a complete *abbacus* text available with a full English translation for the first time. Unfortunately, the advantage offered by having such a fine edition produced by such a reputable publisher is offset by the editor's unfortunate decision to use the youngest and least reliable copy of the text as the basis of the edition. His concomitant decision to relegate the earlier texts to an appendix without the benefits of a standard textual *apparatus* renders the edition largely useless to advanced scholars in the field, while his extensive efforts to justify

the priority of the inferior text burdens the commentary with arcane arguments that will likely mystify and repel any general reader who might be interested in Jacopo's work. The result is a book that satisfies no one.

As is often the case with books in the *abbacus* genre, there are multiple copies of this particular text, all claiming to be written by Jacopo da Firenze at Mons Pesulanus [Montpellier] in the year 1307. Two versions are clearly later copies. Since they are both written on paper, they can be readily dated by watermarks, ink, handwriting, and production styles to the 15th century. One copy now in the Trivulziana Library in Milan (ms.90) is datable to *ca* 1410; the second copy in the Vatican library (Vat. Lat. 4826) dates to *ca* 1450. A third copy, now in the Riccardiana Library in Florence (ms. 2236), is written on vellum and so cannot be precisely dated; but the fact that it uses vellum (which was largely abandoned for writing common texts by the middle of the 14th century), combined with its ink, handwriting, language, and style, make it clear that it was written in the early 14th century, and thus must be accepted as the oldest text.

Of course, chronological priority does not necessarily establish textual priority. A scholar must also examine the details of the text, looking in particular for the kinds of omissions and errors that signal a derivative copy. In this case, the Vatican copy contains many such omissions and errors, all of which can be corrected by looking to the two earlier copies. Indeed, six complete paragraphs/problems found in the earlier Florence and Milan copies are missing from the Vatican version, which shows that its copyist was carefully selecting what he wanted to include. The only novelties in the later copy are the insertion of a short explanatory paragraph in the multiplication tables, a reformulation of six geometry problems, the insertion of an entirely new section on algebra, and the addition of a large set of miscellaneous problems at the end of the book.

None of this is at all unusual. The *abbacus* books as a whole are noteworthy for the variability of their texts. Authors and copyists often took problems, passages, and entire sections from other books without ever giving credit or even noting that fact. The focus was always on learning how to do mathematics and solve problems, not on crediting one's predecessors or preserving 'sacred texts'. Modern

ideas of editing and textual integrity simply did not exist at this time. Compilers felt free to revise, add, omit, or mix their sources in whatever way they wanted. Thus, it is not at all surprising that a 15th-century copyist, while working primarily from an older source, might have omitted some problems that were of no interest to him, added or revised a few paragraphs, and inserted a discussion of algebra, which had become a common feature of other *abbacus* books by that time.

Unfortunately, early in his study of Jacopo's work, the editor became convinced that the Vatican copy, which was clearly written last (he does not dispute this fact at all), nonetheless represents the most authentic text and must be given priority over the two older copies. Elsewhere he reports that he came to this conviction in 1997 when he first examined the algebra section in the Vatican manuscript and noticed how different it was from the traditional presentations of algebra that derived from the tradition of Mohammed bin Musa al-Khwarizmi [Høyrup 2006, 5]. Later comparison with an earlier version of the text that does not contain the algebra led him to believe that the algebra was 'really due to Jacopo' and not a later insertion [2006, 5].

This conclusion was first presented at a conference in Beaumont in 1999, and subsequently published in the proceedings of that conference in 2001 [Høyrup 2001]. Meanwhile, he had published the text of the algebra section alone in *Centaurus* in 2000. His arguments were further elaborated at a second conference in Barcelona in 2003, which were published in *Historia Mathematica* in 2006. Many of the diagrams and discussions found in the present book are taken from these earlier publications, most with few changes.

The grounds that the editor gives for preferring the youngest copy are primarily linguistic and stylistic. I think that he does a better job of presenting them in his first conference paper; but in the current volume he summarizes them by saying,

this [the Vatican] manuscript is very coherent in style as well as regarding the presence of [*sic*] various idiosyncratic features both in the chapters that are shared with [the Milan] and [Florence manuscripts] and in those that are not. [5]

Now style does offer one way to establish a relationship between manuscripts, but it is certainly the least reliable and the hardest to

prove scientifically. This is particularly true for texts in Renaissance Italian, which had not yet become a standard literary language. Indeed, at the time that this text was copied, it can be said that Italian was not yet a language at all but rather a range of local dialects, differing not only from region to region and province to province but even from town to town. Without any standard vocabulary, spelling, or grammar, a copyist might feel free to change the text into the words, forms, and phrases that were familiar to him, or he might stick literally to his exemplar. Several styles could easily become mixed, depending on how attentive he was to his task. The fact that a text seems 'coherent' only indicates that the copyist was being consistent; it says nothing about the state of the original.

Unfortunately, it is clear in hindsight that the editor allowed his first impression of the section on algebra to color all of his subsequent investigations of Jacopo's work and to value weak stylistic impressions above the hard evidence of direct textual comparison. Sadly, he allowed this personal prejudice to affect his entire presentation and treatment of the text, markedly limiting its value to both scholars and the general reader. Thus, the transcription of the younger Vatican text is presented on pages 193–376 as the authoritative text, entirely by itself, with the parallel English translation, even though it is clearly an inferior copy that omits many words and phrases together with the six complete paragraphs that had to be supplied from the Florence and Milan copies. Even a cursory scan shows the large number of omissions that had to be supplied from these earlier copies or corrected by the editor, not to mention the many duplications and insertions made by the Vatican copyist himself. Indeed, while comparing the text with the original, I found that the editor had omitted all of the corrections that the copyist himself made, perhaps because there were so many.

The transcription and translation of this flawed text is then followed by a second transcription of the two earlier texts on pages 383–456, added as if they were an inferior appendage and presented in a very unusual way using different font styles, underlining, and subscripts that is not at all standard for editing variorum texts in modern textual studies. The editor himself calls it a 'semi-critical edition' [379]. I found it extremely difficult to read and essentially useless for scholarly purposes. Moreover, there is no common numbering for the paragraphs or sections of the text, so one cannot readily

compare the texts in the two sections; nor is there a comparative table of contents that would show how the three texts compare in their organization and selection of problems. All of this hinders any effort to examine the editor's hypothesis independently and makes it very difficult for serious scholars of the texts and language of the period to use the book to advance their research in these fields.

But by far the greatest failing of the edition is the complete absence of a glossary, a list of technical terms and their variant forms, a summary list of problems, an index of problem types, a list of coins, or any of the other tools that have become essential to the scholarly presentation of medieval and Renaissance texts. Such *apparatus* have long been standard in the field, and were first applied to an *abbacus* text in Kurt Vogel's edition of the Columbia algorism in 1977, a work that the editor cites and certainly was aware of. In short, the edition lacks the basic standards and tools that are fundamental to modern scholarship in this field.

Other failings clearly follow from the editor's determined effort to establish the priority of the Vatican copy, including exaggerations, misrepresentations, unsupported claims, and a blindness to contrary evidence. For example, on page 6 the editor states that the Vatican manuscript 'is a meticulous (yet not blameless) library or bookseller's copy made from another meticulous copy', when in fact, as we have already seen, it is in fact a very poor copy. It is full of errors, omissions, insertions, and corrections, and is clearly inferior to the two older copies. On page 5, the editor says that

reducible fourth-degree equations were solved routinely in Arabic algebra at least since al-Karaji's time and therefore were no innovation, neither in 1307 nor in the late fourteenth century.

No source is ever given for this very expansive claim, and the editor himself, after an exhaustive comparison with 13 Arabic algebras listed on page 154, not only fails to identify any such source but states, 'We do not know the kind of Arabic algebra that provided him [Jacopo] with his ultimate inspiration' [159], i.e., there is no Arabic source for the equations in the Vatican manuscript.

This fruitless search for a foreign source is driven by the early date of Jacopo's original work, 1307, which places it 20 years before the oldest known vernacular algebra text, that of Paolo Gerardi

[see [Van Egmond 1978](#)]. Since the editor steadfastly maintains that the algebra section found only in the 1450 copy of Jacopo's work is original, he is forced to devote an entire chapter [147–182] to a search for sources that predate Gerardi. In addition to the above-mentioned Arabic texts and Gerardi himself, he looks at several other 14th-century Italian algebras, including those of Dardi da Pisa and Giovanni di Davizzo as well as some anonymous texts now in Parma and Luca. A simple stemma first offered on page 145 grows on pages 167 and 176 to become an incredibly complex nest of manuscripts and links that miraculously leaves the Vatican copy untouched at the top. Clearly some form of Occam's razor ought to apply to textual studies, so that the simplest explanation, in which the Vatican manuscript is a late copy of Jacopo's 14th-century algorism with the insertion of a later algebra section, would be preferred.

And indeed, such a source is readily found. Two late 14th-century algebra texts now in the Biblioteca Nazionale Centrale di Firenze, Fond. Prin. II. V. 152, folios 153r–166r, and Conv. Sopp. G. 7. 1137, folios 110r–111v, give exactly the same equations as the Vatican text in exactly the same order [[Van Egmond 2008](#), 313]. Moreover, they are the common sources for the algebras found in a number of *abbacus* books written in Florence in the middle of the 15th century, which probably stem from the school of maestro Biagio dell'abbaco, who died in 1397, but whose work was carried on by Lucha di Matteo and Calandro Calandri and his sons and students [[Van Egmond 2008](#), 313]. Clearly, the copyist of Vat. Lat. 4826, while revising an old copy of Jacopo's *Tractatus algorismi*, merely inserted a section on algebra that was being widely circulated in his own day, and then further added the additional collection of problems that follow the end of Jacopo's original text. Had the editor not been so firmly wedded to his early conviction that the algebra section in Vat. Lat. 4826 had to be original to Jacopo, he might have discovered this and produced a far better book. As it is, he allowed his initial impression to become a bias that adversely affected the quality of his final work. The result is distorted scholarship, the unwarranted separation of the early and later texts, the lack of a common reference system, and the absence of the standard scholarly *apparatus*, all of which severely reduce the book's value for the serious scholar in this field.

His obsession with proving the authenticity of the Vatican text also diminishes the book's appeal for a more general reader, such as a

student of Renaissance history or someone merely curious about the state of Renaissance mathematics. The title of the book promises a discussion of ‘Early Italian Abacus Culture’, and, as the first edition of an *abacus* book aimed at a broader public, one might have hoped for a general introduction explaining the economic and social background that led to the composition of the *abacus* books, the role that mathematics played in Renaissance society and business, and some illustration of how Renaissance men actually did mathematics, wrote numbers, and solved problems. Unfortunately, anything that might fit this description is limited to about three pages [27–29]. The two-page introduction [3–4] is devoted to a dull review of the scholarly history of the Vatican manuscript, and the description of the three manuscripts that follows [5–25] becomes fully occupied with the editor’s complex linguistic arguments over why the Vatican text must have priority over the other two. The detailed discussion of alternate spellings, words, phrases, and word ratios will bore anyone but the most dedicated student of Italian linguistics. The chapter titled ‘The Abacus Tradition’ [27–44] quickly turns into a detailed summary of the obscure 13th-century *Livero de l’abbecho* and a comparison with the *Liber abbaci*. The long analysis of the mathematical content of the Vatican manuscript that follows this [45–146] is overly technical and will be impenetrable to the general reader. There is no question that this section is an intellectual *tour-de-force*, as the editor displays his wide knowledge of early mathematics by identifying similar problem types in Latin, Greek, Arabic, Indian, and even Chinese problem texts; but it is an effort that will be appreciated only by the most narrow specialist and is already readily available in other well-known sources.⁴

Thus, the editor’s conviction that the youngest manuscript contains the most authentic text has resulted in a severely flawed book. For the serious scholar in the field, it has led him to split the texts, disregard the scientific standards of textual editing, and omit the basic tools of scholarly analysis. For the more general reader or Renaissance scholar, has it buried what could have been a very entertaining and illuminating document beneath a pile of arcane scholasticism. Any attempt to make the book accessible to a more general reader or

⁴ The best is [Tropfke 1980](#), 513–660.

even the wider class of Renaissance scholars was lost in the pursuit of the editor's personal passion.

The overall value of having a complete edition and translation of an entire *abbacus* text available in a quality edition for the first time is undeniable, and this book will retain its value for this purpose alone despite its many other failings. But one can only regret that the editor's fixation on proving his narrow thesis led him to compromise so much else. This book could have been so much more.

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