Mathematics in India by Kim Plofker

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Foreign interest in Indian mathematics has a long history, but it has often been accompanied by puzzlement and frustration. The reaction of the 11th century Muslim astronomer al-Bīrūnī contains many themes which have been echoed by later writers:

> ...even the so-called scientific theorems of the Hindus are in a state of utter confusion, devoid of any logical order, and in the last instance always mixed up with silly notions of the crowd, e.g., immense numbers, enormous spaces of time, and all kinds of religious dogmas.... Therefore it is a prevailing practice among the Hindus *jurare in verba magistri* [to appeal to the word of the master, i.e., to argue from authority]; and I can only compare their mathematical and astronomical literature, as far as I know it, to a mixture of pearl shells and sour dates, or of pearls and dung, or of costly crystals and common pebbles. Both kinds of things are equal in their eyes, since they cannot raise themselves to the methods of a strictly scientific deduction. [Sachau 1992, quoted by Plofker on page 262].

Costly crystals, once recognized as such, were quickly appropriated. Thus, Arabic mathematicians, and then Europeans, adopted the Indian decimal place-value system (the greatest achievement of the Hindus, according to Cajori's *A History of Mathematics* [1919, 88]) and their trigonometric tables (improvements of Ptolemy's chord tables). Once Europeans made direct contact with India, other crystals were found, including evidence that the Hindus knew the binomial theorem 'much better than Pascal' [283]. This last fact came to light too late to influence European mathematics; but it was further evidence of a sophisticated Indian mathematical culture in former times,

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and curiosity about the extent of this culture led to the translation of whole texts from Sanskrit into European languages.[1]

However, by the start of the 20th century, the opinions of western historians of mathematics differed little from those of al-Bīrūnī, 700 years earlier. Smith wrote that

> in the works of all these writers there is such a mixture of the brilliant and the commonplace as to make a judgement of their qualities depend largely upon the personal sympathies of the student. [1923, 152]

Cajori thought that the Indians had climbed to a great height in mathematics (although their actual route was no longer traceable). For example, as well as their decimal system, their algebra too was far advanced of anything that the Greeks had [1919, 83]. (Of course, the Greeks were the standard for what an ancient mathematical culture ought to be like.) Furthermore, the Indians had invented general methods for indeterminate analysis, where Diophantus had used only *ad hoc* methods [1919, 94–95]. But Indian geometry had no definitions, no postulates, no axioms, and no logical chain of reasoning [1919, 86]. In other words, it was not Euclid. Cajori also thought it unfortunate that Indian mathematics had always remained a servant of astronomy, as opposed to its apparently independent existence for the Greeks. Although Cajori appreciated that the Indian habit of expressing their mathematics in verse could aid the memory of someone who already understood the subject, he thought that such verse could only make mathematics obscure and unintelligible to everyone else [1919, 83]. Finally, he claimed [1919, 85] that after Bhāskara II in the 12th century, Indian 'scientific intelligence decreases continually', a sentiment echoed by Smith:

> Mathematics was already stagnant, and the European influence gave it no stimulus. India has always been content to take her time. [1923, 435]

Fifty years later, Boyer presents a similar picture, contrasting the Hindu's 'intuitive' approach with the 'stern rationalism of Greek geometry' [1968, 238]. Indeed, for Boyer, Āryabhaṭa has 'no feeling for logic or deductive methodology' and Brahmagupta, treating

[1] Colebrooke 1817 is an early example.

irrational roots as numbers, displays logical innocence rather than mathematical insight [1968, 232, 242].[2]

It is only quite recently that western writers have tried to understand Indian mathematics on its own terms, to appreciate the context which produced those costly crystals, and to provide the connecting narrative which turns episodes and highlights into a coherent story.

As far as context is concerned, writers such as Cajori and Smith were, to an extent, echoing the mathematical attitudes of their time. Since the mid 19th century, people like Dedekind and Cantor had worked to set mathematics on a firm foundation, independent of physical considerations. Indeed, their aim was to make arithmetic independent of geometry, and separate branches of mathematics soon came to prize their independence of one another. So a culture in which mathematics was so closely entwined with astronomy as Indian mathematics was must have seemed quite backward. Furthermore, a generation or two of mathematicians grew up, many of whom knew nothing of spherical trigonometry or astronomy; and so ancient mathematics embedded in such contexts became difficult to appreciate or even to recognize.

In more recent times, though, scholars have mined new sources, finding new mathematical pearls in non-mathematical rubbish dumps (not just astronomical texts, but also texts dealing with sacred ritual, astrology, or metrical rules for verse). In addition, historians generally have turned away from writing history as a triumphal victor's narrative and have become more open to presenting other participants' points of view. Thus, recent general histories such as Katz' *A History of Mathematics* [1993], have shown more interest both in the mathematics of other cultures and in the problems and contexts which gave rise to mathematics. Like Boyer [1968], Katz devotes a single chapter to the mathematics of India and China; but his text also gives some of the astronomical background needed to appreciate not just Indian, but also Greek and Islamic, trigonometry. Moreover, he makes space for another, more recently discovered, pearl: Mādhava's 14th century discovery of infinite series for the sine, cosine, and arctangent functions, over 200 years before Gregory and

[2] Presumably because Brahmagupta failed to observe the distinction between number and magnitude which, according to Cajori [1919, 93], had retarded the progress of Greek mathematics for 100s of years.

Newton—a pearl which calls into question Cajori and Smith's judgement of a stagnant or declining Indian mathematics after the time of Bhāskara II.

Viewing Indian mathematics on its own terms and providing some narrative structure is probably outside the scope of general texts such as Katz 1993. Their own overall narrative, how global mathematics got to where it is today, needs to concentrate on the main stream of history—Katz devotes three whole chapters to aspects of Greek mathematics, for example—and this probably precludes spending too much time on the smaller streams of other narratives. So these tasks have fallen to other writers.

The fine detail of Indian mathematics continues to be presented through the publication of primary sources and commentaries. One recent example is Keller's *Expounding the Mathematical Seed* [2006], which includes a translation of both Āryabhaṭa's chapter on mathematics and the commentary on this by Bhāskara I.[3] Another is Plofker's own chapter in Katz' sourcebook [2007], which contains excerpts from Indian texts spanning perhaps 2000 years along with brief historical comments and even briefer mathematical explanations. But it is still hard to find an up-to-date, coherent narrative for the history of Indian mathematics; and it is this gap which Plofker tries to fill with the book under review.

Chapter 1 is a short introduction explaining the book's aims, giving a brief history of the Indian subcontinent, and describing the role of Sanskrit, the language in which most of India's mathematical texts are written.

Chapter 2 examines mathematical thought in the earliest Sanskrit texts, the Vedas. These texts are thought to have reached canonical status by about the middle of the first millennium BC. Although the content of the texts is essentially religious, consisting of prayers and descriptions of ritual, they refer to what we now think of as mathematical ideas such as a decimal system of numbering (although not yet a place value system) and factorizing integers. Ritual geometry described in the *Śulba-sūtras* used cords or ropes to solve problems associated with altar shapes and orientation, and included ways of constructing right angles or of transforming rectangles into

[3] [Ed.] See the review in *Aestimatio* by S. R. Sarma [2006].

squares or circles. Plofker discusses attempts to find quantitative ideas in Vedic references to astronomical phenomena. This is one of many controversial topics covered in the book and in each case Plofker gives a brief account of what she calls 'the mainstream narrative', admitting where direct supporting evidence might be lacking (very few documents are more than 400 years old) and mentioning alternative theories which are not quite so mainstream. She is invariably polite towards opposing theories and gives references for those seeking to explore those theories.

Chapter 3 looks for traces of mathematical thinking during the Early Classical Period, which extends from about the middle of the first millennium BC through to the first few centuries AD. It seems that the decimal place value system was adopted during this period, but its origins are obscure. Less obscure, perhaps, are the origins of Indian trigonometry. The incursions of Alexander the Great brought at least northern India into contact with Greek culture, so it may not be too surprising to find Sanskrit verses from this period listing properties of what we now call the sine function. Even here, though, Plofker points out [52] that there is no hard evidence of transmission; and so we can say only that Indian astronomers appear to have been the first to use sines rather than Ptolemy's chords. Mathematical ideas pop up in surprising places, and Plofker shows in section 3.3 how an analysis of metrical structure in poetry can lead to a variation of binary representations.

As already mentioned, astronomy and mathematics are closely interlinked in Sanskrit texts. Chapter 4 provides the necessary background for appreciating these links. Here Plofker explains the basics of geocentric astronomy. With the help of a dozen or so diagrams, she elucidates a series of Sanskrit verses describing how sines can be used to calculate various astronomical parameters. Of particular interest to mathematicians here is the way in which Indians used interpolation techniques to calculate sine values between the standard values. Ptolemy tabulated his chord values at steps of $\frac{1}{2}^\circ$ (360 values between 0° and 180°) but Indian mathematicians recorded just 24 values in steps of 3.75°. This meant that key values could be memorized in verse form. Intermediate values could be then calculated using interpolation techniques which were also remembered in verse form. Of somewhat wider interest perhaps are the uncertain relationships between observations, numerical parameters, and geometric models in medieval Indian mathematical astronomy [120]. As Plofker says, there is still much work to be done here, but the apparent lack of commitment in Indian texts to a particular geometric model for astronomical phenomena seems to place them more in the Babylonian tradition than the Greek. Perhaps this is another situation where the Indians' 'logical innocence' allowed them to experiment in ways which would not have occurred to their European counterparts.

Chapters 5 and 6 deal with the medieval period and the writings of (among others) Āryabhaṭa, Bhāskara I, Mahāvīra, and Bhāskara II. This means that these chapters have a substantial overlap with Plofker's chapter in Katz 2007.[4] Plofker acknowledges this and says that the two accounts are meant to complement one another. As the title of Katz' book suggests, its main purpose is to provide readers with original sources translated into English. There is just enough history and commentary to help readers make sense of these sources. On the other hand, the present book's focus is on building a coherent narrative; so there is significantly more historical background and more commentary, not just on the mathematical meaning of the texts but also on their place in the grand narrative. Knowing that a good proportion of the sources were available in another book, Plofker often simply summarizes the content of a group of verses; and in these cases, I found that it helped to have both books open at the same time, so that the combined texts provided a broad selection of source material and a reasonably full commentary. Space constraints mean that there are still many omissions, but Plofker always indicates where the reader can find a fuller treatment of individual works.

The content of Chapters 5 and 6 is a fascinating portrayal of many aspects of medieval Indian mathematics. We see the emergence of mathematics, if not as a separate discipline, at least as separate chapters on calculation [123] and what we might call algebra [140]. The content of the earliest text devoted solely to mathematics, Mahāvīra's ninth century *Gaṇita-sāra-saṅgraha*, is surprisingly close to medieval European texts such as Fibonacci's *Liber abbaci*, although it also includes topics of particular interest in Sanskrit culture such as the number of poetic meters with a fixed number of syllables [168]. A theme recurring in these chapters is the development of ideas which we might see as being related to calculus, starting

[4] [Ed.] See the review of this book in *Aestimatio* by Clemency Montelle [2007].

from an interest in division by zero [151, 163] that may be useful in astronomy [185, 197], and culminating in Bhāskara's calculation of the area of a sphere [199] by dividing it into regions rather like the segments of an orange (although his actual comparison is with an Indian gooseberry).

The contrasting roles of text and commentary have only recently attracted attention in western mathematics [see Netz 2004],[5] but the verse text and prose commentary format of Indian mathematics formalized this distinction at an early stage. Plofker discusses several examples, including situations where the commentator is the same person as the author and even refers to himself in the third person [190]! Apart from elucidating the mathematical text, commentators also offer higher level views on topics such as, why there are so many rules [190], how it feels to have a clear demonstration, and what makes a good mathematician [198]. The essentially oral culture of dense verse is also fascinating. Are the verses deliberately obscure to test the student's competence [308] or are they sources of fruitful ambiguity [142, 214]? Plofker offers one example [183] of what might be called playful ambiguity from Bhāskara's *Līlāvatī*:

> Those who keep in their throats the *Līlāvatī* having entirely accurate [arithmetic] procedures, elegant sentences, [whose] sections are adorned with excellent [rules for] reduction of fractions and multiplication and squaring [etc.] ...
>
> (Alternative translation:) Those who clasp to their necks the beautiful one completely perfect in behavior, enticing through the delight of [her] beautiful speech, [whose] limbs are adorned by the host of good qualities [associated with] good birth ... attain ever-increasing happiness and success.

Chapter 7 looks at the work of the school of Mādhava in Kerala from about the 14th to the 17th centuries. This focuses mainly on the series expansions mentioned earlier, with a careful discussion both of what the verses attributed to Mādhava actually say, and of the associated commentaries produced by this same school. Explanations and rationales were highly valued in this school, to the extent that some were even rendered in verse [247].

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[5] [Ed.] See the review article in *Aestimatio* by Fabio Acerbi [2005].

In Chapter 8, Plofker discusses Indian interactions with the Islamic world and the struggles that both cultures had in understanding one another, as illustrated in the quotation of al-Bīrūnī at the start of this review [45]. Of particular interest is the question of why Indian mathematics adopted some ideas—for example, after the 12th century detailed tables came to be preferred to the shorter Sanskrit tables which had been memorized in verse form [274]—but not others such as axiomatic deductive geometry [277]. In this connection, I was a bit surprised that there was only limited discussion anywhere in Plofker's book of links with Chinese mathematics, especially as there was mention of Chinese pilgrims returning with Sanskrit texts [181]; but this may be another topic where there is no documentary evidence.

Chapter 9 concludes the main body of the book with a discussion of developments in the modern period, including further interest in clear demonstrations [293] and an account of direct relations with European culture, once again characterized by both interest and mutual misunderstanding.

Two useful appendices introduce the reader to relevant features of Sanskrit language and literature, and list biographical data on 40 or so Indian mathematicians. There is a comprehensive bibliography (over 20 pages) which, along with Plofker's helpful footnotes, should enable the interested reader to look into Indian mathematics in more breadth or depth.

The book is well written and easy to read. There is a good balance of commentary and technical detail, so that a scientifically literate reader can appreciate the overall picture and yet the mathematical reader can still confirm the steps of a representative sample of Indian calculations or explanations. The overall theme of seeing Indian mathematics develop in its own context is well handled, with good discussions of how Indian society, culture, or astronomy are relevant to each mathematical development. I spotted only a couple of minor misprints; and the only irritating feature was the unusual system of bibliographic references which is based on abbreviated names and dates, and occasionally puts names out of strict alphabetic order (because the author's initial took precedence over the next letter of their surname).

There is still much work to be done: there are manuscripts still unread, and paths of development and routes of transmission not understood. But Plofker's book finally offers us, at least in outline, an up-to-date and coherent narrative for the history of mathematics in India.

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