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*Apollonius de Pergé. La section des droits selon des rapports. Commentaire historique et mathématique, édition et traduction du texte arabe* by Roshdi Rashed and Hélène Bellosta

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*Reviewed by*  
Clemency Montelle  
University of Canterbury  
[c.montelle@math.canterbury.ac.nz](mailto:c.montelle@math.canterbury.ac.nz)

The *Cutting Off of a Ratio* is one of the two surviving works of Apollonius (the other being the *Conics*) which is extant only in Arabic. This Arabic text has never before been published. There do exist versions in Latin however: Halley prepared a Latin version entitled *Apollonii Pergaei de sectione rationis libri duo* [1706] and over a century later W. A. Diesterweg published *Die Bücher des Apollonius von Perga „De Sectione Rationis“* [1824], which was based on Halley's Latin edition. More recently, the contents of the work have been investigated in English by E. M. Macierowski (translator) and Robert H. Schmidt (editor) in *Apollonius of Perga, On Cutting Off a Ratio: An Attempt to Recover the Original Argumentation through a Critical Translation of the Two Extant Medieval Arabic Manuscripts* [1988]. This 'critical translation' is based on a critical edition which is yet to be made available to the scholarly community. Selected excerpts from this work have also been translated by Alexander Jones in *Pappus of Alexandria, Book VII of the Collection* [1986, 606–619].

Thus the *editio princeps* of Apollonius' *Cutting off of a Ratio* by Roshdi Rashed along with his collaborator Hélène Bellosta is timely. The work contains a preface, introduction, three preparatory chapters, the Arabic text and French translation on facing pages, notes, an Arabic-French glossary, an index, and a bibliography. The introduction situates the text in the context of Greco-Arabic studies both historically and mathematically. Chapter 1 tackles the mathematical problem covered in this work and the method of analysis and synthesis. Chapter 2 investigates this geometric problem and its breakdown in an algebraic light, and the third chapter concerns the

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history and details of the text including a brief note on whether or not Apollonius was the author of the 22nd proposition. The text and translation takes up the majority of the work and includes an *apparatus criticus* based on two different manuscripts both which date to the early 13th century.

To contextualize this work more firmly, Rashed and Bellosta draw from responses to the text by Pappus of Alexandria (*fl. ca.* AD 320), Ibn Sinān (909–946), and Halley in 1706. At the outset they raise valid questions concerning this work [3ff.]: Why did Apollonius devote so much attention to this particular problem? How are we to read and contextualise such a book in absence of external illuminations? How can we comprehend its structure, characterize its style, and discern the true project of Apollonius? To this end, they present the original text and its translation; but their commentary remains largely based on an algebraic interpretation. Being careful to caution that this approach is worlds apart from the original conception, the algebraic orientation allows them, they maintain, to explore the structure of the work and investigate the systematic character and completeness of the approach of Apollonius. But while one can appreciate, with some effort, the intricacy of this work and its mathematical scope, such an orientation does not directly address the original issues the authors raised at the outset, such as motivation, exposition, and approach in the Greek geometrical context.

Apollonius' work tackles the mathematical problem (as described by Pappus):

How to draw a straight line through a given point to cut off from two given straight lines two sections measured from given points on the two given lines so that the two sections cut off have a given ratio. [Toomer 2008, 188]

The resulting scenarios from this geometrical problem are first solved *via* the classical method of analysis; and then, *via* synthesis, the original proposition is reconstructed. This approach is what makes Apollonius' treatise so important: the systematic presentation of problems worked through *via* Greek geometrical analysis followed by its corresponding synthesis is fascinating for both historians and philosophers of mathematics alike. It was due to this fact alone that the work found appeal amongst such Arab practitioners as Ibn Sinān and Ibn al-Haytham, and no doubt contributed to its existence today. As

Rashed and Bellosta explain, because this geometrical problem was conceived without the notion of metric, Apollonius subdivided the problem into many different configurations to cover every resulting case. These are divided into 21 loci.<sup>1</sup> Each locus is further subdivided where appropriate into 87 distinct ‘incidences’ or cases makes its investigations,<sup>2</sup> 24 of which are covered in book 1 and 63 of which are covered in book 2.

Rashed and Bellosta state that the core of Apollonius’ geometrical discussion can be captured by the algebraic quadratic equation [9]:

$$kx^2 - x[k(a + c) + \epsilon(b - d)] + a(kc - \epsilon d) = 0$$

and show how, for various choices of the parameters  $a, b, c, d, k, \epsilon$ , the resulting cases fall out. This is carefully and methodically done, and the correlation between the geometrical approach of Apollonius and its algebraic rendering by the authors is made more explicit in chapter 2. While this does make the dense Apollonian geometry more tractable, it comes at significant cost. The parallel processes of analysis and synthesis, the very organizing feature of Apollonius’s treatment of each configuration become muted as a result of this algebraic transformation. The documentation and investigation of the details and nuances of these processes in this context remains then for future scholarship. Furthermore, an algebraic examination brings to the fore different properties of each configuration which requires them to be treated in a slightly different order than in the original exposition. Indeed, Rashed and Bellosta’s technical exegesis thus orders and groups the loci as follows: 1–2, 3–7, 8–10, 11–13, 17–21, 14–16.

Rashed and Bellosta note [9] the traditional view that Apollonius wanted this work to be an exemplar for the methods of analysis and synthesis. They themselves argue for a more developed reading. They claim that a more nuanced interpretation can be forwarded, namely, that Apollonius wished to push as far as possible the methods of application of areas using analysis and synthesis to address

<sup>1</sup> Or perhaps 22, with the final being a later addition. See pp. 89–91. The term ‘wad’ in Arabic [13, 469], the equivalent of « τόπος » (Pappus), is translated by ‘lieu’ in French and commonly rendered by ‘locus’ in English.

<sup>2</sup> From the Arabic ‘wuqū’: cf. the Greek « πτωσις ». Like the word ‘case’ (from Latin *casus*), both terms derive from verbs meaning ‘to fall’.

*diorisms* (διορισμοί). This intriguing claim is left somewhat dangling. It is briefly revisited in a footnote [17n3], in which the reader is informed that there are two senses in which the word ‘diorism’ can be interpreted in the Greek mathematical context: the authors cite unreferenced sections in Proclus and identify, without discussion, which one they ascribe to in this context. For elaboration on this key topic in Apollonian studies, I refer the reader to the engaging discussions by Fried and Unguru [2001, 283–306] and by Toomer [1990, lxxxiv–lxxxv].

Also noticeable is the absence of any discussion about the status and importance of this text in the history of transmission in the exact sciences and the critical role Arabic texts play as a means for recovering lost ancient works. This work is a key example and a whole raft of fascinating issues emerge from its existence. What might be some of the circumstances and features of the translation? Were there any conceptual developments that might have occurred as a result of the translation process? How do the circumstances of this work address and develop the themes of naturalization and appropriation as raised by Sabra in his seminal study of 1987 and more broadly by those developed by contributions in Ragep and Ragep [1996]? Transmission never occurs without change and impact, and these would be valuable to consider more deeply, given that this work is a prime example.

Indeed, arcane scholarly skills are needed to handle primary source mathematical manuscripts in Arabic and to produce critical editions, translations, and commentaries on what are frequently challenging and technical treatises. Despite this, there has been a steady stream of eminent publications in this area over the last several decades which have provided valuable and decisive contributions to our understanding of the field and are crucial to its progress. In this respect, one notable absence in this publication is an engagement with the contemporary scholarly community. The lack of acknowledgement of recent research directly related to Apollonius and to this period in the history of mathematics is surprising, particularly given the sentiments the authors express at the beginning on methodology:

les différentes organizations de l’ontologie permettent de saisir  
les différentes strates des sense qui le constituent. [4]

Furthermore, considering that the historical commentary is but a small portion of the book and given the importance of Apollonius in the history of mathematics, this work could have profited immensely from connecting itself more thoroughly and more substantially with recent scholarly literature. However, overall, this work is a valuable contribution to the field. The availability of a critical edition of the text will be a real asset for scholars who will no doubt be as mindful of the work as they are filled with admiration.

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