Numerals and Arithmetic in the Middle Ages by Charles Burnett

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Reviewed by
Jens Høyrup
Roskilde University
jensh@ruc.dk

After *Arabic into Latin in the Middle Ages: The Translators and Their Intellectual and Social Context* and *Magic and Divination in the Middle Ages: Two Texts and Techniques in the Islamic and Christian Worlds*, this third Variorum volume from Charles Burnett's hand collects papers dealing with the period and process of adoption of the Hindu-Arabic numerals. The collection shows us the intricacies of this process, a process which was probably the 'most momentous development in the history of pre-modern mathematics' [IX.15]. Intricacies are certainly not unexpected in a process of this kind; but their precise portrayal can only be painted by someone as familiar as Burnett with the original documents, their languages, their style and context.

Burnett combines this technical expertise with a keen eye for the broader questions to which it can be applied (without which the answers provided by even the best technical expertise can appear naive). It must be said, however, that technical matters and details take up most of the space in the majority of the articles in the volume. The reader with paleographic proficiency will enjoy the many reproductions of manuscript pages.

The volume contains 11 articles of varying length:

- (I) 'The Abacus at Echternach in *ca.* 1000 A.D'. 14 pp. text, 4 pp. reproductions. Originally published in 2002.
- (II) 'Abbon de Fleury, *abaci doctor*'. 11 pp. text, 2 pp. reproductions. Originally published in 2004.
- (III) '*Algorismi vel helcep decentior est diligentia*: The Arithmetic of Adelard of Bath and His Circle', 40 pp. introduction, 37

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pp. edition with translation, 12 pp. edition of *Anxiomata artis arithmetice*, 22 pp. reproductions. Originally published in 1996.

(IV) 'Ten or Forty? A Confusing Numerical Symbol in the Middle Ages'. 7 pp. text, 2 pp. reproductions. Originally published in 2008.

(V) 'Indian Numerals in the Mediterranean Basin in the Twelfth Century, with Special Reference to the "Eastern Forms"'. 32 pp. text, 20 pp. reproductions. Originally published in 2002.

(VI) 'The Use of Arabic Numerals Among the Three Language Cultures of Norman Sicily'. 4 pp. text, 7 pp. reproductions. Originally published 2005.[1]

(VII) 'Why We Read Arabic Numerals Backwards'. 6 pp. text. Originally published in 2000.

(VIII) 'The Toledan *Regule* (*Liber alchorismi*, part II): A Twelfth-Century Arithmetical Miscellany' (in collaboration with Ji-Wei Zhao and Kurt Lampe). 8 pp. introduction, 34 pp. text edition, 33 pp. translation, 16 pp. mathematical translation and notes. Originally published in 2007.

(IX) 'Learning Indian Arithmetic in the Early Thirteenth Century'. 10 pp. text, 2 pp. reproductions. Originally published in 2002.

(X) 'Latin Alphanumerical Notation, and Annotation in Italian, in the Twelfth Century: MS London, British Library, Harley 5402'. 10 pp. text, 5 pp. reproductions. Originally published in 2000.

(XI) 'Fibonacci's "Method of the Indians"', 11 pp. text. Originally published in 2005.

The recurrent themes are summed up in the short preface [vii], according to which the volume

> brings together articles on the different numeral forms used in the Middle Ages—actually from the 10th through the 13th century—and their use in mathematical and other contexts. Some articles study the introduction of Hindu-Arabic numerals into Western Europe between the late 10th and the early 13th centuries, documenting in more detail than anywhere else the different forms in which they are found, before they

[1] Greatly reduced in size: the footnotes are *ca* 5 pt. This should have been reset in spite of Variorum's normal principles.

> acquired the standard shapes with which we are familiar today [articles I, V, VI, VII, VIII, IX, XI]. Others deal with experiments with other forms of numeration within Latin script, that are found in the twelfth century: e.g., using the first nine Roman numerals as symbols with place value [III], abbreviating Roman numerals [IV], and using the Latin letters as numerals [X]. Different types of numerals are used for different purposes: for numbering folios, dating coins, symbolizing learning and mathematical games, as well as for practical calculations and advanced mathematics. The application of numerals to the abacus [I, II], and to calculation with pen and paper (or stylus and parchment) is discussed [VII, IX].

As reflected in these words, Hindu-Arabic numerals were indeed not adopted merely because they happened to present themselves; they came together with practices (astronomy, astrology, commerce) where they served. For a long while it was not obvious that all of these practices were best served by the complete Hindu-Arabic system and not by one of the alternatives that were tried: that is, by counters inscribed with the Hindu-Arabic numerals used on an abacus board emulating the place value system (the 'Gerbert' abacus)—a place value system using Roman numerals 'I' through 'IX' instead of the unfamiliar Hindu-Arabic shapes—or by a Latin emulation of the Greek alphabetic notation. Nor was the shape of the Hindu-Arabic numerals clear and certain from the start, since those who adopted them initially were in contact with different regions of the Arabic world that used different styles.

In detail, article I describes a large parchment sheet from the Benedictine monastery of Echternach from *ca* AD 1000 that carries the earliest extant specimen of what has been known as a 'Gerbert' abacus. As pointed out by Burnett [I.92],

> nothing precise is known about the origin of this device but our testimonies rather associate a revival of its use with Gerbert d' Aurillac, especially with his period as a teacher at Reims (972 to 983).

According to Burnett, it

> seems likely that Gerbert introduced the practice of marking the counters with Arabic numerals (which he would have come across when he studied in Catalonia, before coming to

> Reims), and established a form of the abacus board that became an exemplar for most subsequent teachers of the abacus.

This assumption has the advantage over a presumed invention from scratch that it creates harmony between pre-Gerbertian references to the abacus and the ascriptions to Gerbert. As argued by Burnett, the Echternach abacus agrees so well with the description of Gerbert's own abacus made by his pupil Richer and with Bernelinus' prescriptions for its use that we may reasonably regard it as a faithful copy of Gerbert's own board. However, as noted, another apparently contemporary manuscript from Echternach[2] —'virtually a facsimile' [I.101]—may contain what is in itself an even more faithful copy but to which complementary commentaries have been added, commentaries which explain, among other things, how to calculate with Roman duodecimal fractions (a vestige of earlier medieval monastic computation not represented on the original Gerbert abacus as described and copied in the two manuscripts described here but soon fitted onto the board in three extra columns). The parchment sheet itself as well as the quasi-facsimile enumerate the three-column groups by means of Arabic numerals (in abacus shape), thus making obsolete Walter Bergmann's observation [1985, 212] that no positive evidence supports the traditional belief that the 'Gerbert' abacus made use of these already from the beginning.[3]

Article II raises the question whether the mathematical honor of Gerbert's contemporary Abbon de Fleury can be saved. Nikolaus Bubnov [1899, 203] concluded from the paucity of substance in the references to the abacus that we have from Abbo's hand that his competence on the instrument on which he declared himself a doctor was

[2] Now MS Trier, Stadtbibliothek 1093/1694.

[3] Thus Burnett's polite report of Bergmann's stance. Actually, Bergmann's claim is much stronger (though based on very weak evidence), namely, that the late-10th-century abacus used counters carrying Greek letter-numerals and that the first use of Hindu-Arabic numerals on the counters is to be dated two generations after Gerbert; common use according to Bergmann belongs to the second half of the 11th century. This is now not only obsolete but directly falsified.

Article III, written earlier, still follows Bergmann (in the weak version) and accepts the claim that the earliest appearance of the Hindu-Arabic numerals on abacus counters is in the pseudo-Boethian Geometry II [III.227 with n28]. This of course has to be corrected in view of article I.

quite restricted (unless the lines where this occurs were added by a copyist). Burnett goes through the evidence (including references to Abbo in manuscripts from pupils of his citing his teaching) and finds that all of it is concerned with the mystical properties of numbers and not at all with technical teaching. The lack of mathematical substance thus does not prove his incompetence; nor, it must be said, is any evidence for particular skill supplied by the sources.

Article III

> investigates the kind of arithmetic practised by Adelard of Bath, his colleagues, and his immediate successors. This will lead us to re-examine the introduction of the algorism into Europe and, incidentally, to make some comments on the terminology for, and use of, the zero, and on the authorship of the Latin versions of Euclid's *Elements* known as Version I and Version II. The key texts are Adelard's passage on arithmetic in his *De eodem et diverso*, his *Regulae abaci*, the versions of Euclid's *Elements* associated with the name of Adelard of Bath, glosses to Boethius' *Music* which mention Adelard, glosses to Boethius' *Arithmetic* in the same manuscript as those to Boethius's *Music*, the *Helcep Sarracenicum* of H. Ocreatus, and the contents of [a] Coventry manuscript [containing another copy of the latter text]. [III.222f]

As far as the early *De eodem et diverso* and *Regulae abaci* are concerned, the analysis substantiates what was already pointed out by Marshal Clagett [1970, 61f], namely, that they show no influence from the Arabic world. The analysis of sources connected to the various versions of the *Elements* leads Burnett to conclude that Version 1 'seems to be a direct translation from the Arabic made by Adelard himself (probably with the help of an arabophone)' [III.229],[4] whereas Version 2 is indeed an ongoing (branched) project rather than a single version:[5] evidence is offered that friends and/or students of Adelard were involved in the project while he was still alive.

The article is accompanied by an edition and translation of the *Helcep sarracenicum*, whose title means 'Saracen calculation' ('helcep', as it is argued, rendering Arabic 'al-ḥisāb'), and which explains

[4] Busard, in his edition of the text, did not feel able to determine the authorship [1983, 18].

[5] This is in agreement with Busard and Folkerts 1992.

the place value system and how to calculate within it. Remarkably, the whole treatise represents the digits by Roman, not Hindu-Arabic, numerals—a pretty exemplification of how the new numerals and place value system represented a double difficulty, and that it could therefore be judged adequate to introduce one of them without the other.[6] The treatise was dedicated to Adelard and, hence, written during his lifetime—and also, it appears, before its genre acquired the standard name 'algorism'. Burnett suspects its author (an otherwise unidentified 'Ocreatus', whose name appears, however, in various puns in writings from the same intellectual environment) to have been more competent than Adelard in Arabic and, hence, perhaps involved in the production of Version 1.

Article IV deals with a particular writing of '40' as a ligature 'X^{L}', often reduced (perhaps by scribal misunderstanding) to a mere 'X'. The origin of this ligature is in Visigothic script. Analyzing all mathematical and astronomical/astrological manuscripts where it is used,[7] Burnett reaches the conclusion that it occurs in particular in John of Seville's earlier translations—Seville later used Hindu-Arabic numerals—and that his use of it seems natural, since the ligature was in common use in his environment. Plato of Tivoli and Raymond de Marseille also employ it, even though it was probably foreign to the places where they worked (Barcelona and Marseille, respectively); they can be presumed to have been influenced by John's writings. Use of the ligature by Gerard of Cremona in his translation of the *Almagest* (where Roman numerals are employed) is doubtful. Other 12th-century translators based in Aragon and Navarra but coming from elsewhere seem not to have used it (unlike John, indeed, they had not been brought up with it). In general, as formulated in the conclusion [IV.87], Burnett maintains,'When Hindu-Arabic numerals finally prevailed among mathematicians, the ligature disappeared altogether.'

The first part of article V presents the two principal ways to write Hindu-Arabic numerals, 'Eastern' and 'Western', together with the intermediate Palermitan way (on which more below). A table

[6] The terminology is also in debt to earlier abacus writings and to the Boethian tradition.

[7] The manuscripts, though not autographs, appear to reflect the originals faithfully.

shows their shapes in 53 manuscripts and on two coins (9 Arabic, 4 Greek, the rest Latin, dating from the 10th to the 13th century). The second part concentrates on the appearances of the Eastern type in Latin manuscripts. It finds that this type turns up in a few manuscripts that point back to Hugo of Santalla.[8] It is possible that Hugo's inspiration comes from manuscripts once belonging to the Banū Hūd library in Zaragoza. Manuscripts going back to Hugo's friend Hermann of Carinthia also use it (but here the Eastern form seems to be what the scribe is accustomed to himself). The earliest manuscript of the version of the *Elements* made directly from the Greek also uses the Eastern form.

However, all these manuscripts were probably written in Tuscany, which leads Burnett to Abraham ibn Ezra, who came from the region where Hugo and Hermann worked but whose essential work in the present respect—the Pisan Tables (if they really are his) and explanations of how to use them—were also written in Tuscany. The Eastern forms are also used in these commentaries. Still, after weighing the complete evidence Burnett comes to the conclusion that

> the use of Eastern forms in the Latin texts associated with Abraham ibn Ezra is probably due...not so much to Abraham himself as to his Latin associates, who were using the tables of Pisa. The combined testimony of these manuscripts strongly indicates that the Eastern forms were being used in Pisa and Lucca in the mid-twelfth century. [V.251]

Thus, even the Eastern forms used in the Hugo- and Hermann-manuscripts may say little about what the originals did. As pointed out, Pisan external connections were oriented at that moment toward Antioch and Constantinople—and even Greek writers using Hindu-Arabic numerals initially used the Eastern forms (the Western forms only turn up in 1252).

An appendix lists and describes 26 Latin manuscripts using Eastern and Palermitan forms.

The short article VI at first describes the particular character of the translations from Norman Sicily, where translations were made from the Greek as well as from the Arabic into Latin, and where some

[8] Since the last copyist has difficulty in understanding them, he at least cannot have introduced the Eastern Hindu-Arabic numerals.

scholars at least knew all three languages; and notes the consequence that translations from the Greek were sometimes supplemented by Arabic material (thus the translation of the *Almagest* as well as of Euclid's *Optics*). After that, it describes the particular Palermitan forms of the Hindu-Arabic numerals—forms intermediate between the Eastern and the Maghreb style, as is the Arabic script of a trilingual psalter prepared at the Norman court. Burnett suggests as a common explanation that the Arabic scribes of the royal chancery (an institution perhaps emulating the chancery of the Egyptian Fatimids) had been taught in Egypt, but where the characters they had learned at home differed too much from those locally used (which were in Maghreb style) they adopted the latter.

Article VII, also short, discusses why (e.g.) 'twelve' is written '12' and not '21'. Initially, it is pointed out that there are two reasons for this. Firstly, this is the way in which the number is written in Arabic, where lower orders of magnitude are written first in the right-to-left reading direction; secondly, Greek alphabetic as well as Roman numerals write the higher orders to the left. However, as Burnett points out, the direction to be used was none the less uncertain at first and in need of explanatory justification. Early algorisms often speak of the position to the left as 'later' (perhaps translating an Arabic text directly), and when presenting the numerals in sequence they have '9' to the left (as Arabic texts would have it). By the early 13th century, according to Burnett, most algorisms had adopted what we would consider the normal orientation; but he points to a short algorism probably written shortly before 1250 where 'before' is still to the right.[9]

Article VIII is an urgently needed 'working edition' of the *Regule*, a miscellany of arithmetical texts glued to the *Liber alchorismi*,[10]

[9] According to the two editions [Curtze 1897, 2; Pedersen 1983, 176], Sacrobosco's *Algorismus vulgaris* also considers the position to the right as 'first' and gives the sequence of numerals as '9 8 7 6 5 4 3 2 1'. Even Jacopo da Firenze, in some debt to Sacrobosco but not copying, still has the sequence '10 9 8 7 6 5 4 3 2 1' or '0 9 8 7 6 5 4 3 2 1' in his Tuscan *Tractatus algorismi* of 1307, and his opinion about what is 'first' and what is 'last' is unstable [see Høyrup 2007, 196–202, 385, with Høyrup 2009, 117 for correction].

[10] The existing edition was made from one manuscript by Baldassare Boncompagni [1857b, 93–136]. André Allard, in his edition of the *Liber alchorismi*, only refers occasionally to a 'seconde partie' [1992, xvii, xix, xxxviii–xl]

made from what Burnett and his co-authors (Ji-Wei Zhao and Kurt Lampe) consider the best manuscript (Paris, BNF lat. 15461) and followed by English and mathematical translations.

The *Regule* consist of seven distinct textual elements, to which come multiplication tables for the orders of sexagesimal fractions and for the numbers 1 through 9, and a magic square. From the totality of manuscripts, the authors conclude that they were put together in Toledo (whence the name they give to the whole, 'Toledan regule'). They also point out an affinity with the *Liber mahamaleth* and with Gundisalvi's *De divisione philosophiae*.

The contents of the *Regule* cover various arithmetical rules concerning progressions, multiplication and division; abstractly formulated rules for the conversion of metrological units; the rule of three[11] and the partnership rule; the rules for the three mixed algebraic second-degree cases; and rules for finding a hidden number. Finally, there is a philosophical/numerological justification of the principles of Hindu-Arabic reckoning.

The treatise shares with the *Liber mahamaleth* (edited by Vlasschaert [2010]) as well as with the *Liber abbaci* the inscription of numbers for a calculation within a rectangular frame, probably corresponding to a dust- or clay-board (*takht* or *lawḥa* respectively).[12] Although the overlap in contents between the three treatises is limited, it cannot be neglected; and the *Regule* thus casts light on the environment that produced the two larger treatises.[13] For, since the algebra of the *Regule* is not taken from al-Khwārizmī (neither from known translations nor from the Arabic original), it can no longer be taken for granted that the lost algebra chapter of the *Liber mahamaleth*—and, for that matter, the algebra to which Abū Bakr

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without ever explaining in any way what these words refer to. Actually, this second part is identical with the *Regule*, present in all but one of the 10 manuscripts.

11 Understood as the answer to a riddle, not as a real-life commercial problem: somebody, 'concealing from you the fourth number', asks.... Obviously, the author is a scholar and not a clerk or a merchant school teacher.

12 Fibonacci speaks of it as a *tabula*: see Boncompagni 1857a, 118.

13 We should not forget that one of the earliest manuscripts of the *Liber abbaci* [Vatican, Pal-Lat. 1343, new foliation 47r] refers to a *magister castellanus* as the source for chapter 9, 'On Barter'.

refers in the *Liber mensurationum* [see Busard 1968]—is identical with al-Khwārizmī's text. Any further study of these three texts (and many others until the 14th century) should henceforth take the *Regule* into account.

Article IX analyses two short introductions to an algorism, all three items to be found in a manuscript that also contains the *Helcep Sarracenicum*. They represent an intermediate stage of the development of the algorism genre, preceding the kind of codification achieved by Alexandre Villedieu's *Carmen de algorismo* and Sacrobosco's *Algorismus vulgaris* in the earlier decades of the 13th century[14]—part of the terminology is still inherited from the operations on the Gerbert abacus, and one of the commentaries applies to the abacus just as well as to algorism.

Article X deals with the

> use of the Latin letters in their alphabetic order as numerals, on the model of the notation for numerals which is normal in Greek, Arabic and Hebrew. [X.76]

This notation was not widespread. Indeed, Burnett locates it 'in a group of closely related works written by a certain "Stephen" and

[14] Burnett [IX.15] states that

> the acceptance of the algorism within the canon of European mathematics was ensured by the magisterial *Liber abbaci* of Leonardo of Pisa (Fibonacci)...and the more popular manuals of Alexander of Villa Dei...and of John of Sacrobosco.

I must object, however, that there is no evidence in favour such a role for Fibonacci. Apart from a barely possible reference to his solution to a problem about purchasing a horse in Jordanus of Nemore's *De numeris datis* [II.27 see Hughes 1981, which shares the numerical parameters with Boncompagni 1857a, 245–248 but speaks of the method as 'Arabic'], no school mathematician before Jean de Murs appears to have made use of or even known the *Liber abbaci*—and Jean uses the algebra and the treatment of roots, not the algorism [see l'Huillier 1990, 12].

Note 1 states that the texts of the *Carmen de algorismo* and the *Algorismus vulgaris* are available only in Halliwell 1841. Actually, a working edition of the *Carmen* is in Steele 1922, 72–80, while working editions of the *Algorismus vulgaris* are in Curtze 1897, 1–19 and Pedersen 1983, 174–201, the former based on a single manuscript, the latter on 4 manuscripts with control of 11 more (including the one used by Curtze).

an "'Abd al-Masīḥ of Winchester"', two of which are dated 1121 and 1127, respectively, and which were both copied in Antioch. Stephen was from Italy and appears to have written for an Italian public. However, Burnett's article concentrates on the manuscript British Library, Harley 5402, where a planetary table using this notation is accompanied by a key, showing that its users were not expected to know the notation. These notes, written in a mixture of Italian and ungrammatical Latin, mention the date 1160 and refer to the tables of Lucca, which were derived from the above-mentioned Pisan tables. Since Abraham ibn Ezra, involved in these, had been in Lucca in the 1240s, it is suggested as a possibility, but not asserted explicitly, that the annotations might go back to Abraham.

From the linguistic point of view, the manuscript is important since it contains one of the earliest known examples of writing in Tuscan.

Article XI deals with a never-discussed puzzle contained in the oft-quoted introduction to Fibonacci's *Liber abbaci.* Fibonacci states that his father wanted him to stay and be taught 'for some days' in a 'calculation school'[15] in Bejaïa, where he was introduced to the 'art [of calculation] by the nine figures of the Indians'. The knowledge of this art pleased him so much that he learned all that he could about how it was studied in Egypt, Syria, Greece, Sicily, and Provence when going there for the sake of trade. But (this is the puzzle) he also writes:

> I reckoned all this, as well as the [Latin] algorism and the arcs of Pythagoras [the Gerbert abbacus] as a kind of error as compared to the method of the Indians.

As Burnett protests, both the Gerbert abacus and the algorism were also based on the nine figures of the Indians, and these were known by Latin scholars since the mid-12th century.[16] The algorithms

[15] XI.87n1 'Genitor meus...studio abbaci per aliquot dies stare voluit et doceri'. Burnett translates 'studio abbaci' by 'abbaco school' thereby intimating an institution of the same kind as is found in Italy a century later. While this may be an unwarranted jump if taken to the letter, the word 'doceri' ('be taught') at least guarantees that 'studio'must be taken in the meaning of 'school'.

[16] Regarding Syria etc., Burnett points out that Fibonacci says nothing about the Indian figures being used there and states that

for computing were not the same in the three cases, he admits, but of course they had common features. So, is Fibonacci just showing off or self-advertising (the Indians being in odor of ancient wisdom)? This is Burnett's closing hypothesis.

This is indeed possible: we know that Fibonacci's use of references was strategic—he says nothing about his indubitable debt to existing Latin translations from the Arabic (al-Khwārizmī's *Algebra* [see Miura 1981] as well as Abū Bakr's *Liber mensurationum* [see Høyrup 1996, 55]). However, there is no reason to believe that Fibonacci speaks about the Hindu-Arabic numerals only. Indeed, the preface, as translated by Burnett, continues thus:

> Therefore, concentrating more closely on this very method of the Indians, and studying it more attentively, adding a few things from my own mind, and also putting in some subtleties of Euclid's art of geometry, I made an effort to compose, in as intelligible a fashion as I could, this comprehensive book, divided into 15 chapters, demonstrating almost everything that I have included by a firm proof, so that those seeking knowledge of this can be instructed by such a perfect method (in comparison with the others), and so that in future the Latin race may not be found lacking this (knowledge) as they have done up to now.

Apart from the Euclidean material and some unspecified contributions made by Fibonacci himself, the whole of the *Liber abbaci* was thus considered to present 'this very method of the Indians'. However, already on page 24 of the 459 pages of the Boncompagni edition we are introduced to the notations for ascending continued fractions and

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> the most common forms of numerals used by merchants in the Mediterranean in the Middle Ages were derived from Greek alphanumerical notation.

However, whatever was done in commercial interaction and for accounting and notarial purposes does not reveal much about what was done when calculation was practised as an 'art'. Ibn Sīnā, as he says in his autobiography, was taught the use of Hindu numerals by a greengrocer [see Gutas 1988, 24], thus by a merchant, not by an astronomer or professional mathematician (which in the context amounted to much the same). In general, different purposes called for the use of different notations [see Rebstock 1993, 12; Rebstock 2008 27–29].

other composite fractions invented in the Maghreb or al-Andalus during the 12th century, notations totally unknown (according to extant documents, including the *Liber mahamaleth*) in the Latin world but used systematically and heavily by Fibonacci. Later there also follows a huge amount of 'practical arithmetic' (by far exceeding what was needed in commercial practice, of course), and even an algebra that goes well beyond what was known through the translations of Robert and Gherardo. According to Fibonacci's words, all of this belonged under the heading 'method of the Indians'. Much of it can be found in the *Liber mahamaleth*. But nothing suggests that Fibonacci knew that book; thus, he was entitled to believe that the Latin race had up to now been 'lacking this knowledge'.

The question remains why Fibonacci characterizes it as the 'method of the Indians'. He may, as Burnett proposes, just be self-advertising. But we should take note of his understanding that the whole subject matter of his book (the Euclidean and personal additions excluded, probably also chapter 15, part 1) constituted a single complex. This complex encompassed much material known not only from Arabic writings but also from Sanskrit mathematicians presenting and using the methods of 'the world'.[17] We know nothing about how the commercial community carried this knowledge structure between India and the Mediterranean but we may be sure that it did. Somehow, it may have been known in the environment that it was connected to India—or this may have been concluded mistakenly by Fibonacci because the complex encompassed 'Indian' numerals. Self-advertising remains a plausible explanation but alternatives are at hand (and one need not exclude the other).

To sum up, this collection of articles is immensely rich in insights—often so detailed that the reader may have to work through an article several times in order to get all the points. Often, by necessity, the conclusions drawn are tentative—but when they are, this is always made explicit; only rarely is it possible to suggest a more likely interpretation of the sources than what is proposed by Burnett. The book can be recommended to anybody working on the matters

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[17] This distinction between scholarly and 'lay' mathematics is made by Bhāskara I [see Keller 2006, 1.7, 12, 107f].

which it deals with; but it can also be recommended that the reader go to the richness of its text with patience.

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[18] This contains upwards of 60 printing errors: the editor did not realize that computer conversion should be followed by proof reading.

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